Random Walk and ARIMA Models in prediction of salary development in the Czech business sphere

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Abstract

The aim of this study is to provide an outline of the development of the average gross wages and wage medians of the Czech business sphere up to the end of 2018 using an optimised ARIMA model with a constant and an optimised random walk model (RW). It will be shown that both models will provide relatively comparable predictions, on the basis of which interested subjects will be able to execute their adaptive expectations. Utilising the common basis for the year 2005, consumer price indices (CPI) will be used for the confrontation of the development of average gross wages and wage medians with the development of Czech inflation. From the graphic outputs it will be clear that the expected trend in the growth of Czech average wages and wage medians, expressed with the aid of indices with the same basis, will grow more rapidly than the expected trend in the growth of Czech inflation.

Keywords: random walk, ARIMA, average wage, wage median, consumer price index

JEL classification: E24

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1. Introductory assumptions

For the requirements of the analysis we shall consider the time series of the average gross monthly wage (AGMW) and monthly wage medians (MWM) with quarterly frequency, published by Information System on Average Earnings (ISPV) and also the time series of consumer price indices (CPI) with quarterly frequency, published by the Czech Statistical Office (CZSO). All the time series considered start with the 1st quarter of 2002 and end with the last quarter of 2010. The methodological approach used will be that of authors Box and Jenkins (1970) for the modelling of time series, especially the ARIMA model and the random walk model (RW). On the basis of these models we shall outline the development of the AGMW, MWM and CPI up to the end of 2018. This development will be important in particular for the requirements of interested subjects, who will then be able more easily to create their adaptive expectations (for more information see Husek, 2007 or Evans, Honkapohja, 2001). So that it will be evident in the future what will be the status of the development of AGMW and MWM in comparison with the development of Czech inflation, the indices of AGMW and of MWM will be calculated on the basis of the average for 2005, and they will show that it is possible to expect that the rising trend of AGMW and MWM will be more rapid than the growth of inflation. The goal of this thesis is to show that the Czech business sector need not, providing

ceteris paribus, directly fear the devaluation of wages through inflation, as the growth of inflation is expected to be slower.

Stochastic modelling may seem to be a simple instrument, nevertheless in the dynamic world of a developing economy, including that of the Czech Republic, the assumptions of even far more complicated models may be very easily infringed. The past shows us that, in spite of the course of the economic crisis of 2008 (see Jerabkova, Zeleny, 2011), there have not been any changes so dramatic that they would strongly alter the development of the trend in AGMW and MWM. The decline was only temporary, not permanent (see e.g. Zeleny, 2001). The constant increase in the aggregate price level in time and with it the constant rise in AGMW and MWM can be assumed for the future with roughly the same probability as the rise in fuel prices and other economically precious raw materials.

2. Modelling of AGMW and MWM

On the basis of the methodological approach of the authors Box and Jenkins (1970) we identified the model ARIMA (1, 1, 1) with constant for the times series "AGMW" and further the RW with drift (see for instance Hughes, 1996), where the drift was optimised at the value 273.457. The estimates of the parameters of the ARIMA (1, 1, 1) model with constant are given in Tab. 1.

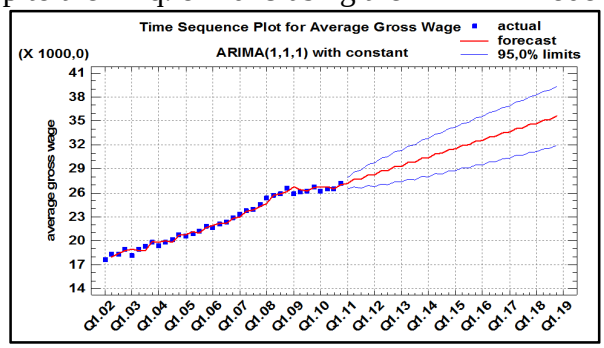
Table 1. Parameters of the ARIMA (1, 1, 1) with C for the times series "AGMW"

Parameter	Estimate	Stnd. Error	T-Statistic	P-value
AR(1)	-0.982392	0.0573603	-17.1267	0.000000
MA(1)	-0.807264	0.134941	-5.98235	0.000001
Mean	266.198	54.5818	4.87704	0.000028
Constant	527.708			

Source: own constructions

The diagnostic tests of the model indicate that the non-systematic component of the model is not auto-correlated, it is homoscedastic and has normal distribution (see for instance Husek, 2007). Using the ARIMA (1, 1, 1) model with constant and the RW with a drift of 273.457 we calculated the predictions up to the end of 2018, which are depicted in Fig. 1 and 2. It is evident that both models provide almost comparable results, albeit the intervals of reliability in the ARIMA model are somewhat narrower.

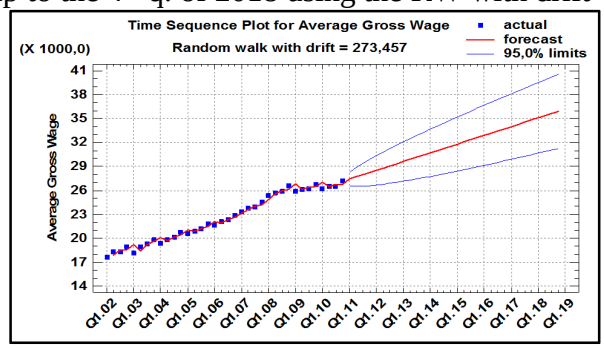
Figure 1. Development of AGMW (in CZK) from the 1st q. of 2002 to 4th q. of 2010 with calculated predictions up to the 4th q. of 2018 using the ARIMA model



Source: ISPV, own constructions

The use of MWM is suitable for expression of the wage differentiation of the Czech Republic, as more than 60 % of the population is already receiving a below-average wage. A wage median is a more robust statistic, which is less encumbered by remote values.

Figure 2. Development of AGMW (in CZK) from the 1st q. of 2002 to 4th q. of 2010 with calculated predictions up to the 4th q. of 2018 using the RW with drift



Source: ISPV, own constructions

In a similar way to that used in the case of the time series "AGMW" we identified the ARIMA (2, 1, 2) model with constant and further the RW with drift for the time series "MWM". The drift was optimised at the value of 212.171. The estimated parameters of the ARIMA (2, 1, 2) model with constant are given in Tab. 2.

Table 2. Parameters of ARIMA (2, 1, 2) with C for the time series "MWM"

Parameter	Estimate	Stnd. Error	T-Statistic	P-value
AR(1)	0.39407	0.15897	2.4789	0.019025
AR(2)	-0.646481	0.16629	-3.88766	0.000519
MA(1)	1.02827	0.143553	7.16302	0.000000
MA(2)	-0.887883	0.107661	-8.24701	0.000000
Mean	195.766	51.4061	3.80822	0.000645
Constant	245.179			

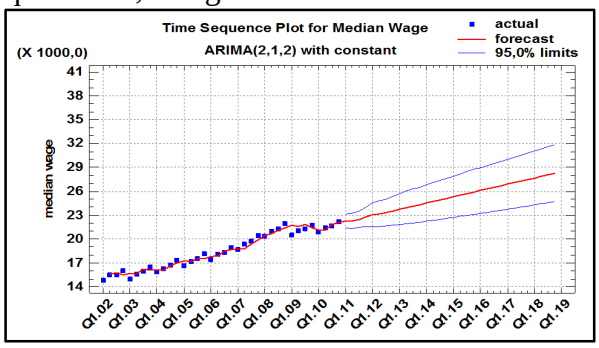
Source: own constructions

This time, too, the diagnostic tests of the model are statistically significant on the 5% level of significance. Using the ARIMA (2, 1, 2) model with constant and the RW with a drift of 212.171 we calculated predictions up to the end of 2018, which are depicted in Fig. 3 and 4.

3. Modelling of CPI

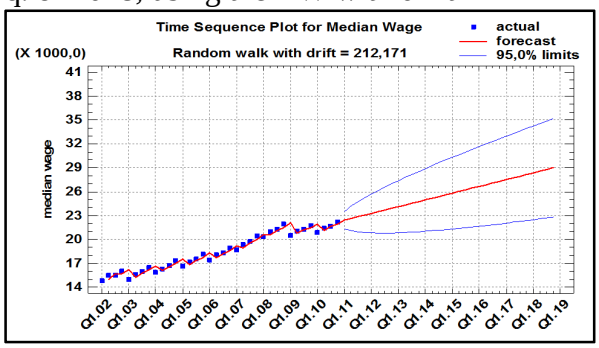
The CPI is generally recognised as the measure of inflation in economics. At present the indices published are based on the average for 2005. The inflation trend is an important index in the individual expectations of economic subjects. The expected level of inflation has to be used in the indexing of wages, in the estimation of the valuation of long-term orders and also in the provision of loans and credit. In the case that the future level of inflation were to grow

Figure 3. Development of MWM (in CZK) from 1st q. of 2002 to 4th q. of 2010 with calculated predictions up to the 4th q. of 2018, using the ARIMA model



Source: ISPV, own constructions

Figure 4. Development of MWM (in CZK) from 1st q. of 2002 to 4th q. of 2010 with calculated predictions up to the 4th q. of 2018, using the RW with drift



Source: ISPV, own constructions

more rapidly than the growth of AGMW or the MWM, the real growth of wages would halt and become a real drop. Wages would still increase, but only nominally. In the long term the economic subjects in the business sector would be able to purchase less and less goods and services from their wages.

For the next part of the analysis we identified the model ARIMA (0, 2, 1) with constant and also the RW with drift for the time series "CPI". The drift was optimised at the value of 0.564103. The estimates of the parameters of the ARIMA (0, 2, 1) model with constant are given in Tab. 3.

Table 3. Parameters of the ARIMA (0, 2, 1) with C for the time series "CPI"

Parameter	Estimate	Stnd. Error	T-Statistic	P-value
MA(1)	1.07262	0.0499598	21.4697	0.000000
Mean	0.00142017	0.00204462	0.69459	0.491771
Constant	0.00142017			

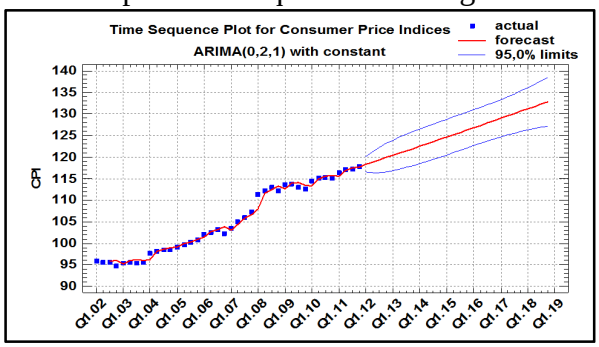
Source: own constructions

The diagnostic tests of the model indicate that the non-systematic component of the model is not auto-correlated, is homoscedastic and has normal distribution. Using the model of ARIMA (1, 1, 1) with constant and the RW with a drift of 273.457 predictions were calculated up to the end of 2018 and these are shown in Fig. 5 and 6.

4. Comparison of the development of the CPI with the development of AGMW and MWM

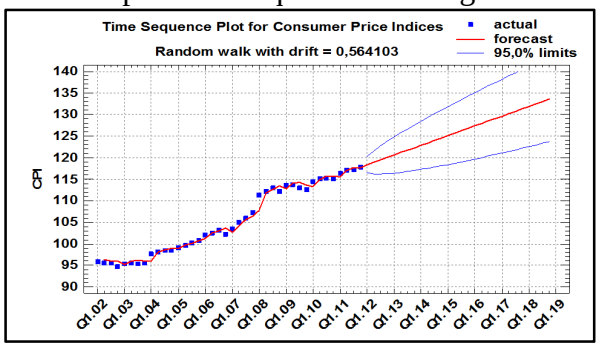
For the requirements of the comparison of the development of the CPI with the development

Figure 5. Development of CPI (basis – average for 2005 = 100) from 1st q. of 2002 to 4th q. of 2010 with prediction calculated up to the 4th q. of 2018 using the ARIMA model



Source: CZSO, own constructions

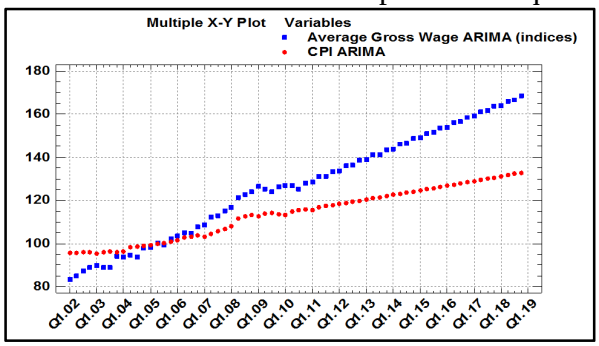
Figure 6. Development of CPI (basis – average for 2005 = 100) from 1st q. of 2002 to 4th q. of 2010 with prediction calculated up to the 4th q. of 2018 using the RW



Source: CZSO, own constructions

of AGMW, or with the development of MWM, an average was calculated from the values for the observations from the 1st quarter of 2005 – 4th quarter of 2005 and on this basis were calculated the indices of AGMW or the indices of MWM. These indices, obtained from the ARIMA model and the RW, were confronted with the values of the CPI in a single graph, with the confrontation of the development of AGMW indices from the ARIMA model being in Fig. 7 and the confrontation of the development of these indices from the RW in Fig. 8. The confrontation of the values of the CPI with the values of the indices of MWM is shown in Fig. 9 for the ARIMA model with constant and in Fig. 10 for the RW.

Figure 7. Development of AGMW indices and CPI up to the 4th q. of 2018, ARIMA with C

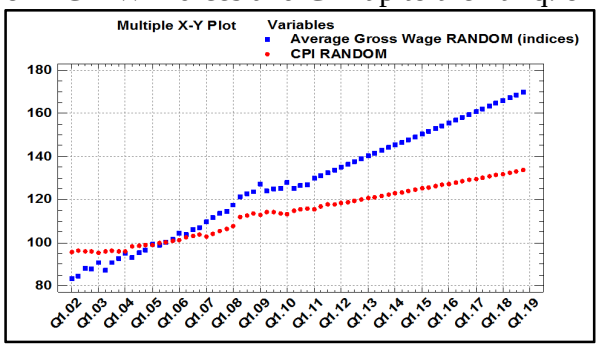


Source: ISPV, CZSO, own constructions

5. Conclusion

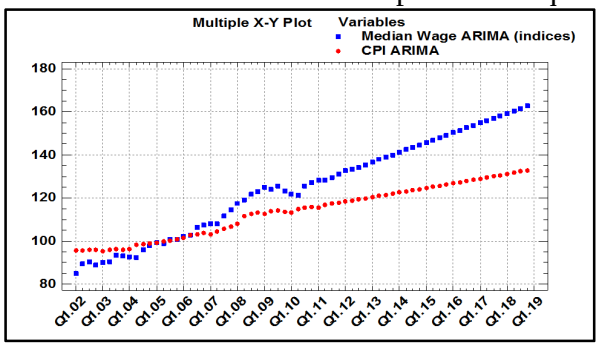
The assumption mentioned in the introduction is evident from the outputs. Assuming ceteris paribus it is to be expected that the AGMW and MWM in the business sector of the Czech Republic will rise. The CPI or aggregate price level of the domestic economy will also rise. Due to the point that the speed of the rising trend of AGMW and MWM might be higher than the rate of growth of inflation, it is possible that real wages may continue to grow in the business sector in the future. There is not institution in the Czech Republic that would attempt to compare the future evolution of CPI and wages, using sophisticated statistical techniques. The limitations which must be accepted are based on the assumption *ceteris paribus*. In the future will not happen anything, what would dramatically changed the current trend. In the future we would like to perform another study involving for example the theory of expectations from business tendency surveys.

Figure 8. Development of AGMW indices and CPI up to the 4th q. of 2018, RW



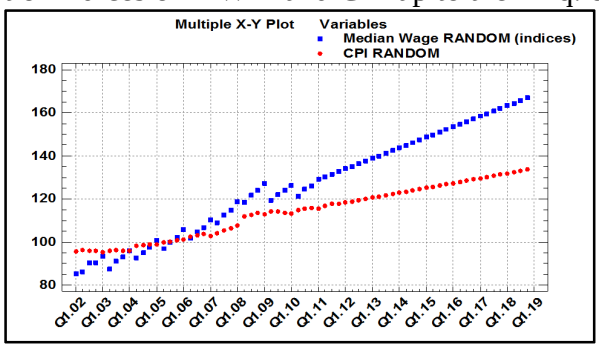
Source: ISPV, CZSO, own constructions

Figure 9. Development of indices of MWM and CPI up to the 4th q. of 2018, ARIMA with C



Source: ISPV, CZSO, own constructions

Figure 10. Development of indices of MWM and CPI up to the 4th q. of 2018, RW



Source: ISPV, CZSO, own constructions

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