THE IMPACT OF REGIONAL MIGRATION ON POTENTIAL FUTURE SIZE AND STRUCTURE OF THE SELECTED CZECH DISTRICTS: THE APPLICATION OF THE COMPLETE PREDICTION MODEL

Ondrej Simpach

University of Economics Prague

ABSTRACT

Regional demographic projections are important for predicting the future age and sex structure of specific population, and the construction of regional projections is particularly difficult because it is a small population, for which they may not be sufficiently detailed data and this can lead to a significant distortion. In the case of the Czech Republic and its regions, there is no lack of data, as well as level statistics for the regions are high. Apriori assumptions that enter into the demographic projections regarding mortality, fertility and migration, affect the outcome of the more, the populations are smaller and the horizon of the projections is longer. Computational system for construction projections, which was created for the purpose of this analysis, is able to accept any apriori assumption and create so many different projection scenarios. On some of these assumptions there will be demonstrated several regional projections, the data will be considered for all regional cities, with the exception of the City of Prague. The City of Prague will be a reference population and provide us the certain assumptions which will be applied to regional cities.

JEL: *J11*

KEYWORDS: *regional demographic projection, mortality, fertility, migration, regional city*

INTRODUCTION

Under certain circumstances, the population projections help us to estimate the characteristics of particular demographic structures in the future. This requires the use of several important demographic assumptions. Component projection method without migration is not practical for the real analysis, because it considers the same level of fertility and mortality in the future, as it was at the beginning of the projection. This assumption is not correct, because we know that the probability of death of *x*-years old person decreases in time, what leads to an increase the life expectancy. The aging in the top of the demographic tree is caused by the increasing life expectancy, and it is necessary to incorporate to population projections. It also cannot be assumed the constant level of fertility, because women change their mind on the family and its size over time. In the case, that the projections consider a constant level of fertility only, the results would be misleading, because we can generally expect that women in procreative age will change their preferences in the next few decades.

One of the biggest mistakes may be the omission of migration. The migration of economically active population out of the district (and the continuation of this trend) may cause a decrease of the labour force in the district in the future, and this district will not be lucrative for business subjects that are creating jobs. A strong trend of migration (e.g. economically active men) to the district can continue to cause, that these men at the end of their productive lives stay and grow old in the district. Then there will be a high concentration of seniors, for whom the social care system of this district should not be prepared.

The aim of this study is to provide an estimate of the demographic structures of the selected districts of the Czech Republic using the assumptions of decreasing probability of death of *x*-years old persons, changes in fertility and using the expected migration trend in these districts as well. The population projections will be constructed with the horizon of 50 years with all of these assumptions. For estimating the regional demographic age and sex structures, there is very often used the component method (see Fiala, (2002)), which is supplemented by certain assumptions for mortality, fertility and expected migration. In the present study there will be created the population projections in two scenarios for all regional cities of the Czech Republic, (with the exception of the City of Prague), because Prague is considered as the reference category with the lowest total fertility rate in the country to which the each of regional cities will limit in the projection horizon. Projection will be made from the threshold of the 1st January 2011 and its horizon will be 50 years. Regional cities which will be considered for this analysis are:

- Brno,
- České Budějovice,
- Hradec Králové,
- Jihlava,
- Karlovy Vary,
- Liberec,
- Olomouc,
- Ostrava,
- Pardubice,
- Plzeň,
- Ústí nad Labem and
- Zlín

and partial assumptions that will be used in the analysis will arise from the further methodology.

1 METHODOLOGY AND ASSUMPTIONS

For the purpose of individual analyses of districts there will be used the complete life tables for males and complete life tables for females from the Czech Statistical Office (CZSO) database for the period 2006–2010, the range of tables will be 0–104 of completed years of life. (105 years is ω, which is the age at which any person from the initial population is not alive yet). These tables will be converted to the abridged life tables of males and abridged life tables of females for the period 2006–2010 and 0-year-old persons will be considered separately. The range of tables will be therefore 0, 1–4, 5–9, 10–14, 15–19, …, 100–104 of completed years of life. Let us now for each year t the numbers of living set on

$$l_{t,0} = l_{t+5,0} = l_{t+10,0} = \ldots = 100\,000\,,$$

where t = 2008, 2013, 2018, …, 2063. Using the coefficient of decrease of probability of death q_x we can for each year t calculate the numbers of living

$$l_{t,x} = l_{t,x-h} \times \left(1 - k \times \frac{l_{t-5,x-h} - l_{t-5,x}}{l_{t-5,x-h}}\right), \qquad (1)$$

where k is the coefficient of decrease of probability of death and h is width of age interval (5 years). Coefficient of decrease of probability of death will be selected 0.92 for males and 0.93 for females and it was calculated on the basis of knowledge and outputs mentioned by

Dotlačilová et al. (2012). Now that we have the numbers of living l_x in abbreviated life tables for the years t = 2008, 2013, 2018, ..., 2063, now we are able to calculate the numbers of living in the centres of the time intervals 2011–2015, 2016–2020, 2021–2025, ..., 2056–2060. We perform the calculation by linear interpolation

$$l_{t,x} = \frac{4 \times l_{t-1,x} + l_{t+4,x}}{5}. \tag{2}$$

Using the numbers of living in life tables we are now able to calculate the life expectancy at birth, and we can use the relationship, mentioned e.g. by Keyfitz (1964)

$$e_0^0 = \frac{\sum_{x=0}^{110} l_x - \alpha \times l_0 + (\alpha - 0{,}5) \times l_1}{l_0}, \tag{3}$$

where α is the proportion of the lower elementary file of deaths (CZSO recommends setting the value of 0.85). Due to abbreviated life tables, we can use a simpler formula, which is for the purpose of this study adapted to form

$$e_{0,t}^0 = h \times \frac{\sum_{x=1-4}^{100-104} l_{t,x} - \frac{l_{t,1-4}}{2}}{l_{t,1-4}}, \tag{4}$$

where h is width of age interval (5 years). For the continuation of the analysis it is necessary to calculate the projection coefficients and the projection coefficients will be calculated separately for live-born persons, separately for 0–4 years old persons and separately for 5–9 years old and older. Given that the analysis consider the future decrease of the probability of death q_x (which is incorporated in the earlier calculated numbers of living), the projection coefficients will include this decline for particular time intervals 2011–2015, 2016–2020, 2021–2025, ..., 2056–2060 as well. Projection coefficient for live-born persons and the time interval t may take the form, which is used by Koschin (1993)

$$P_{t,*} = \frac{(1-\alpha) \times l_{t,0} + (2+\alpha) \times l_{t,1} + 2 \times l_{t,5}}{5 \times l_{t,0}} \tag{5}$$

and for abridged projection tables of this analysis can be used

$$P_{t,*} = \frac{(1-\alpha) \times l_{t,0} + (2+\alpha) \times l_{t,1-4} + 2 \times l_{t,5-9}}{5 \times l_{t,0}}. \tag{6}$$

Projection coefficient for 0–4 years old persons and the time interval t has the form

$$P_{t,0-4} = \frac{2{,}5 \times (l_{t,5} + l_{t,10})}{(1-\alpha) \times l_{t,0} + (2+\alpha) \times l_{t,1} + 2 \times l_{t,5}} \tag{7}$$

and for abridged projection tables of this analysis can be used

$$P_{t,0-4} = \frac{2{,}5 \times (l_{t,5-9} + l_{t,10-14})}{(1-\alpha)\times l_{t,0} + (2+\alpha)\times l_{t,1-4} + 2\times l_{t,5-9}}. \tag{8}$$

Finally, we can calculate the projection coefficients for persons 5–9 years old and older, using formula

$$P_{t,x} = \frac{l_{t,x+5} + l_{t,x+10}}{l_{t,x} + l_{t,x+5}} \quad \text{for} \quad x \geq 5-9 \tag{9}$$

and for abridged projection tables of this analysis can be used

$$P_{t,x-(x+h-1)} = \frac{l_{t,(x+5)-(x+5+h-1)} + l_{t,(x+10)-(x+10+h-1)}}{l_{t,x-(x+h-1)} + l_{t,(x+5)-(x+5+h-1)}} \quad \text{for} \quad x \geq 5-9. \tag{10}$$

For the projection of live-born we need to use apriori assumptions. Total fertility rate can be either estimated for individual analysed population (see Langhamrová, Fiala, (2009)), or modelled (see Bozick, William Bell, (1987)). For the analysed districts of the Czech Republic it is possible to use both of the assumptions or some other. Calculation tables can be recalculated with any assumptions and for the purposes of this study there will be used the assumption, that the total fertility rate of the analysed districts will linearly approximate to the total fertility rate of reference district – to the City of Prague in 2060, (where the total fertility rate is the lowest in the country, 1.36 children per woman in 2006–2010). We will assume that the regions can imitate this trend. Specific fertility rates for the district of the City of Prague in 2006–2010 is calculated like

$$f_{x,2006\text{-}2010}^{PRG} = \frac{N_{x,2006\text{-}2010}^{(v),PRG}}{5 \times S_{x,2008}^{(F),PRG}}, \tag{11}$$

where $N_{x,2006\text{-}2010}^{(v),PRG}$ are the numbers of live-born of *x*-years-old mothers (where *x* = 15 to 49 completed years of life) in the City of Prague in the period 2006–2010 and $S_{x,2008}^{(F),PRG}$ are the numbers of women *x*-years-old, (where *x* = 15 to 49 completed years of life) in the City of Prague in 2008 and the total fertility rate is the summation of all the specific fertility rates with consideration of width of the age interval *h* (5 years)

$$tfr_{2006\text{-}2010}^{PRG} = 5 \times \sum_{x=15-19}^{45-49} f_{x,2006\text{-}2010}^{PRG}. \tag{12}$$

Then we can calculate the specific fertility rates for each district in 2006–2010 by equation

$$f_{x,2006\text{-}2010}^{district} = \frac{N_{x,2006\text{-}2010}^{(v),district}}{5 \times S_{x,2008}^{(F),district}}, \tag{13}$$

where $N_{x,2006-2010}^{(v),district}$ are the numbers of live-born of *x*-years-old mothers (where *x* = 15 to 49 completed years of life) in the analysed district in the period 2006–2010 and $S_{x,2008}^{(F),district}$ are the numbers of women *x*-years-old, (where *x* = 15 to 49 completed years of life) in the analysed district in 2008 and the total fertility rate in this district is

$$tfr_{2006-2010}^{district} = 5 \times \sum_{x=15-19}^{45-49} f_{x,2006-2010}^{district}. \tag{14}$$

The first we calculate specific fertility rates at the end of the projection (years 2056–2060), then we will focus on specific fertility rates in the middle of the projection (years 2031–2035). Specific fertility rates at the end of the projection period 2056–2060 are

$$f_{x,2056-2060}^{district} = \frac{tfr_{2056-2060}^{district,ex.}}{tfr_{2006-2010}^{PRG}} \times f_{x,2006-2010}^{PRG} \tag{15}$$

where $tfr_{2056-2060}^{district,ex.}$ is the expected level of total fertility in the considered district in the years 2056–2060, (in all of the analysed districts will be considered the current level of total fertility rate in the City of Prague). Expected total fertility rate in the analysed district in the middle of the projection (in years 2031–2035) is

$$tfr_{2031-2035}^{district,ex.} = tfr_{2006-2010}^{district} - \frac{1}{2} \times \left(tfr_{2006-2010}^{district} - tfr_{2056-2060}^{district,ex.} \right) \tag{16}$$

and the specific fertility rates in the middle of the projection (in the years 2031–2035) are

$$f_{x,2031-2035}^{district} = \frac{tfr_{2031-2035}^{district,ex.}}{tfr_{2006-2010}^{PRG}} \times f_{x,2006-2010}^{PRG}. \tag{17}$$

By linear interpolation we are now able to calculate the specific fertility rates for each time period before the middle of the length of projection as

$$f_{x,t}^{district} = f_{x,2006-2010}^{district} + \frac{\left(f_{x,2031-2035}^{district} - f_{x,2006-2010}^{district} \right)}{(2031-2006) \times (t-2006)} \tag{18}$$

where *t* are the time periods 2011–2015, 2016–2020, 2021–2025 and 2026–2030 and *h* is the length of the interval (5 years). We also calculate all specific fertility rates for each of time period after the middle of the length of projection as

$$f_{x,t}^{district} = f_{x,2031-2035}^{district} + \frac{\left(f_{x,2056-2060}^{district} - f_{x,2031-2035}^{district} \right)}{(2056-2031) \times (t-2031)} \tag{19}$$

where *t* are the time periods 2036–2040, 2041–2045, 2046–2050 a 5051–2055 and *h* is the length of the interval (5 years).

The analysis will also include the migration and important variable for us will be the increment of people by moving. Component method of population projection with a simplified migration model uses e.g. Langhamrová, Fiala (2012), modelling of migration uses in his study Kale et al. (2005). For presented study we find out what was the trend in migration in the last 5 years for which we have data. This trend stayed constant in all analysed districts. So the question arises: will be in the next 50 years possible to keep the same trend of population growth (or decrease), as in the last 5 years for which we have data? Some districts may be, but rather this assumption will not apply. Some of districts had in recent years a significant outflow of population, the other districts had a significant inflow of population. Maintaining the outflow of migrants would cause that in next 50 years in the district would live almost no one, maintaining the inflow of migrants would cause that the size of the district would become double or triple.

One of the assumption, which is used in many natural sciences for setting in equilibrium, is the exponential decline. Exponential decline is possible to observe in physics and chemistry (stabilization of reflections of tennis ball or foam of sparkling drinks), also in part of economic and statistical sciences is used for determining the durability of equipment, for solving queuing theory and theory of graphs (see Berge (1962)), or for getting equilibrium in econometric models in the analysis of impulse response (see Pesarana et al. (1998). The deflection of model from the equilibrium by unit impulse is monitored, and quality model is able after sufficiently short time return to equilibrium with exponential decay.

In addition to the constant migration trend will therefore be mentioned a second assumption, which will be presented in the analysis. It is the assumption of exponential decline between the beginning and the end of projection. The expression of this decline can be obtained using a modification of the density of probability of exponential distribution as

$$MIGR_t^{district} = \lambda e^{-\lambda t}, \qquad (20)$$

where t is the time period and λ is suitable parameter. Using the total share of migrant males and migrant females from the last five-year observed data, we will try to estimate the relative proportions of migrant males and migrant females in the age of x-years (i.e. relative age structure of migrants). Individual indices for age of x-years can be described as

$i_x(p)^M$ for males, respectively

$i_x(p)^F$ for females.

From the assumptions of a constant trend of the migration to the future or from the assumptions of exponential decline of migration trend we will determine the expected numbers of migrants enm_t^{M+F} (males and females together) for time period t = 2011–2015, 2016–2020, ..., 2056–2060. The numbers of migrants in a time period *t*, completed *x*-years of age and sex simply determine as

$$MIGR_{x,t}^{M} = enm_t^{M+F} \times i_x(p)^M \text{ for males, respectively} \quad (21)$$

$$MIGR_{x,t}^{F} = enm_t^{M+F} \times i_x(p)^F \text{ for females.} \quad (22)$$

The actual population projection will be create with a threshold of 1st January 2011, taking as input data for each analysed districts we can use the abbreviated demographic structures by age and sex in a 5-years age intervals (h = 5) at 31st December 2010, because the initial states of year t are equal to final states of year t–1. We can use the approach listed e.g. by Fiala (2002), which we can modify to formula

$$S_{x,t}^{(M),district} = N_{t-h}^{(v),(B),district} \times P_{*,t-h}^{(M),district} + \frac{\left(MIGR_{x,t-h}^{(M),district} \times \sqrt[3]{P_{*,t-h}^{(M),district}}\right)}{2} \text{ for } x = 0-4 \quad (23)$$

for males and analysed district and

$$S_{x,t}^{(F),district} = N_{t-h}^{(v),(G),district} \times P_{*,t-h}^{(F),district} + \frac{\left(MIGR_{x,t-h}^{(F),district} \times \sqrt[3]{P_{*,t-h}^{(F),district}}\right)}{2} \text{ for } x = 0-4 \quad (24)$$

for females and analysed district. The states of older people, males and females in the analysed district we can calculate using the equation

$$S_{x,t}^{(M),district} = S_{x-h,t-h}^{(M),district} \times P_{x-h,t-h}^{(M),district} + \frac{\left(MIGR_{x-h,t-h}^{(M),district} \times \left(P_{x-h,t-h}^{(M),district}\right)^{\frac{2}{3}} + \sqrt[3]{MIGR_{x+h,t-h}^{(M),district}}\right)}{2} \quad (25)$$

for $x \geq 5-9$, for males and analysed district, respectively using the equation

$$S_{x,t}^{(F),district} = S_{x-h,t-h}^{(F),district} \times P_{x-h,t-h}^{(F),district} + \frac{\left(MIGR_{x-h,t-h}^{(F),district} \times \left(P_{x-h,t-h}^{(F),district}\right)^{\frac{2}{3}} + \sqrt[3]{MIGR_{x+h,t-h}^{(F),district}}\right)}{2} \quad (26)$$

for $x \geq 5-9$, for females and analysed district. Given that the projection tables should began empty from the top (because no live-born persons have entered to the projection tables yet), we will calculate the numbers of live-born persons for the time period t = 2011–2015, 2016–2020, ..., 2056–2060. Using formula

$$N_{x,t}^{(v),district} = 5 \times \left(\frac{\left(S_{x,t}^{(F),district} + S_{x,t+h}^{(F),district}\right)}{2}\right) \times f_{x,t}^{district} \text{ for } x = (15-19)-(45-49) \quad (27)$$

we calculate the numbers of live-born to *x*-years old mothers for considered time period and analysed district and then the total numbers of live-born boys for this time period and district as

$$N_t^{(v),(B),district} = \sum_{x=15-19}^{45-49} N_{x,t}^{(v),district} \times 0{,}515 \tag{28}$$

and the total number of live-born girls for this time period and district as

$$N_t^{(v),(G),district} = \sum_{x=15-19}^{45-49} N_{x,t}^{(v),district} \times 0{,}485\,, \tag{29}$$

where the proportions of boys (0.515) and girls (0.485) at birth provides e.g. Pavlík et al. (1986).

2 PERFORMED EXPERIMENTS

Using mentioned apriori assumptions and the formulas written above, there was prepared computing system, where the regional demographic projections were calculated. Demographic projections consider the decrease of probability of death q_x (respectively increase the life expectancy e_x^0), the trend of total fertility rate in districts incline to the total fertility rate of the City of Prague (linear trend) and two migration scenarios. In most cases, the constant trend of migration to the future seems to be unrealistic and unsustainable. Either high immigration would increase in the subsequent 50 years the population to double or even triple size, or in the opposite case the high emigration in the subsequent 50 years would cause the disappearance of these populations.

Left pictures always show the demographic projection of the analysed district with constant trend of migration, right pictures show the demographic projection of the analysed district with an exponential decrease of trend in migration. In the Fig. 1 we can see the projected demographic and sex structure of Brno, in Fig. 2 České Budějovice, in Fig. 3 Hradec Králové, in Fig. 4 Jihlava, in Fig. 5 Karlovy Vary, in Fig. 6 Liberec, in Fig. 7 Olomouc, in Fig. 8 Ostrava, in Fig. 9 Pardubice, in Fig. 10 Plzeň, in Fig. 11 Ústí nad Labem and finally in Fig. 12 Zlín.

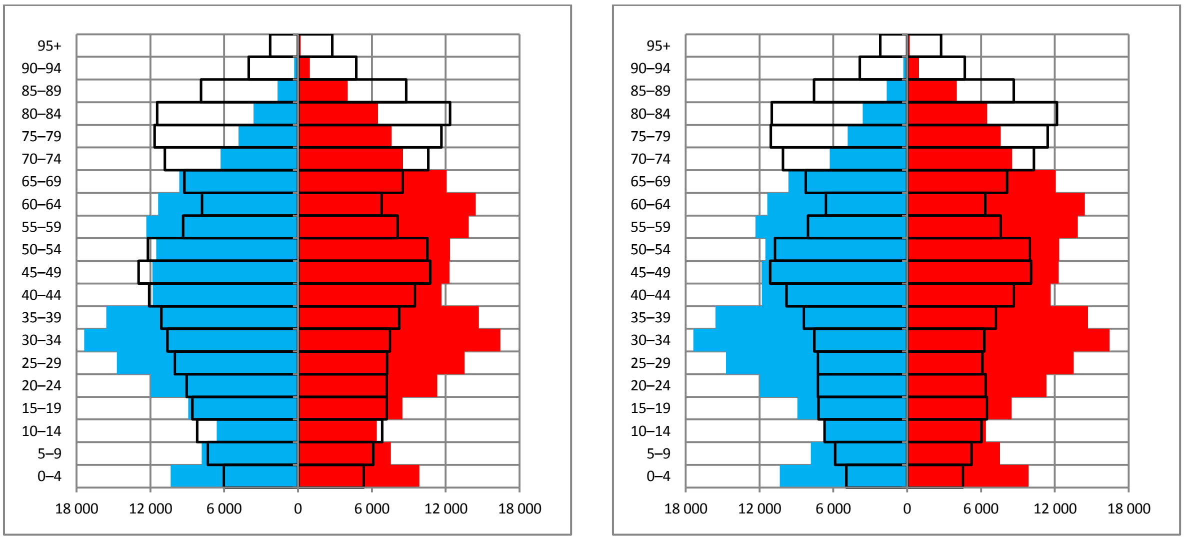

Fig 1: Demographic structure of district Brno at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

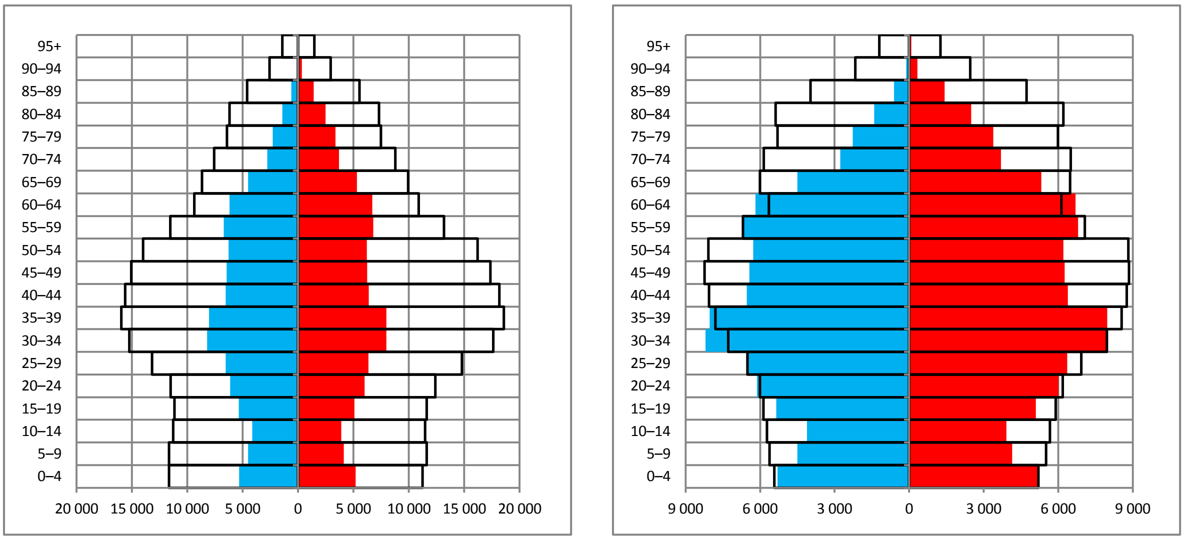

Fig 2: Demographic structure of district České Budějovice at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

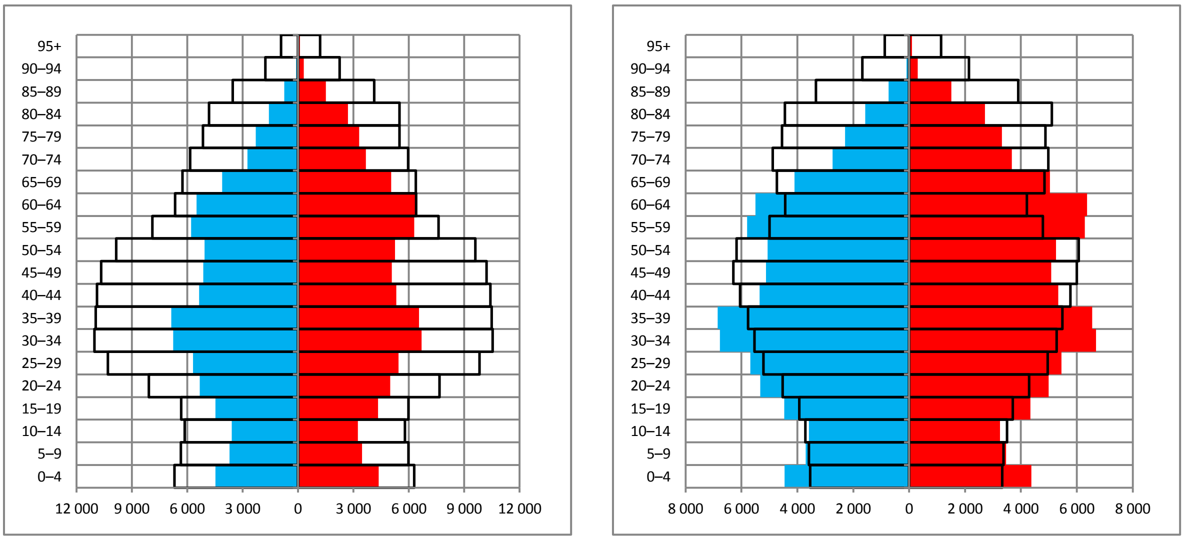

Fig 3: Demographic structure of district Hradec Králové at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

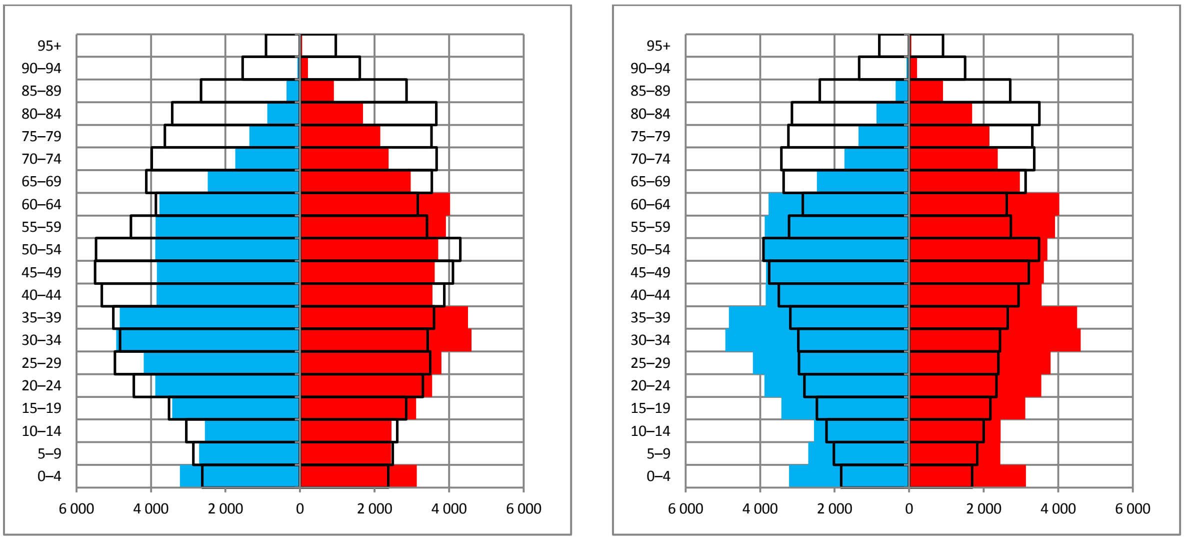

Fig 4: Demographic structure of district Jihlava at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

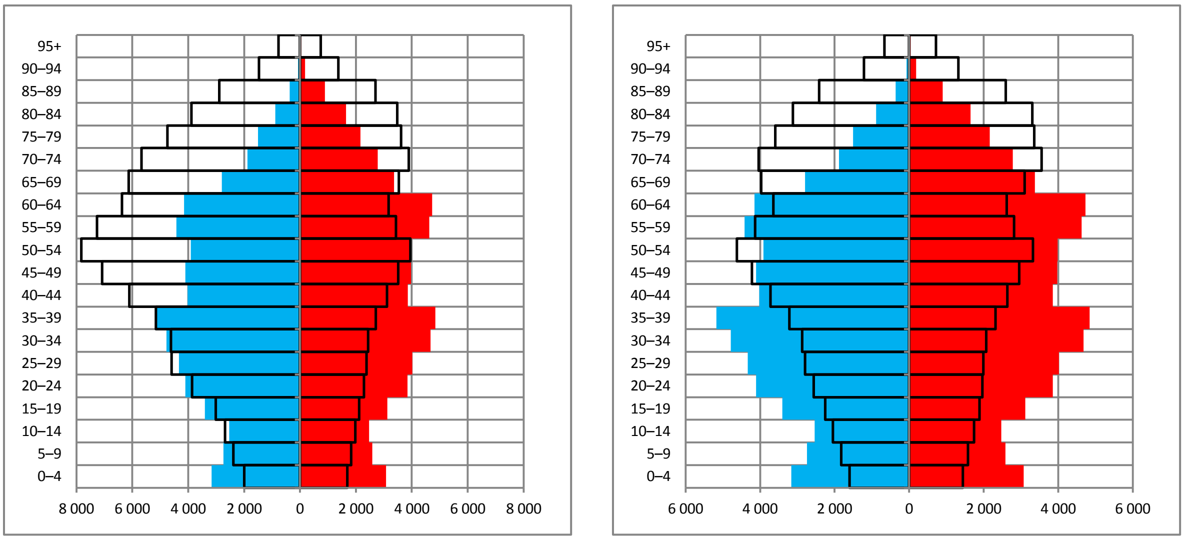

Fig 5: Demographic structure of district Karlovy Vary at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

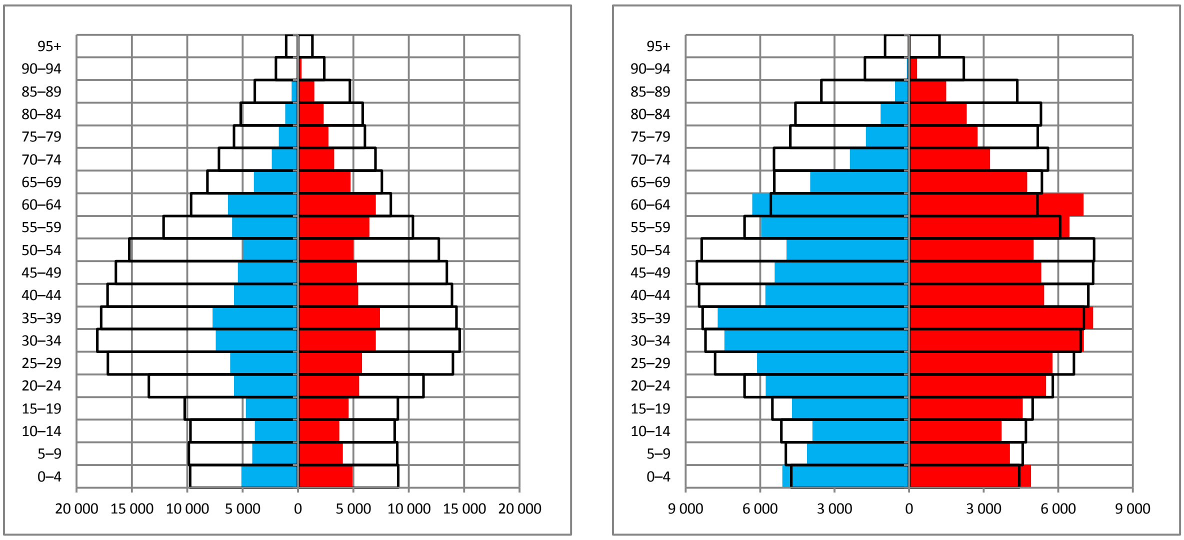

Fig 6: Demographic structure of district Liberec at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

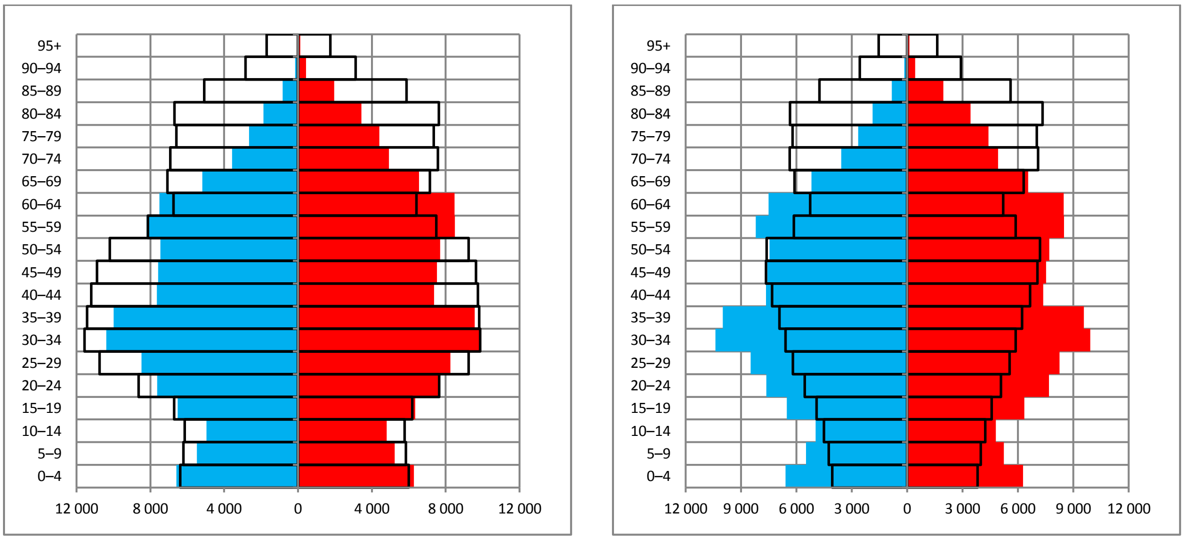

Fig 7: Demographic structure of district Olomouc at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

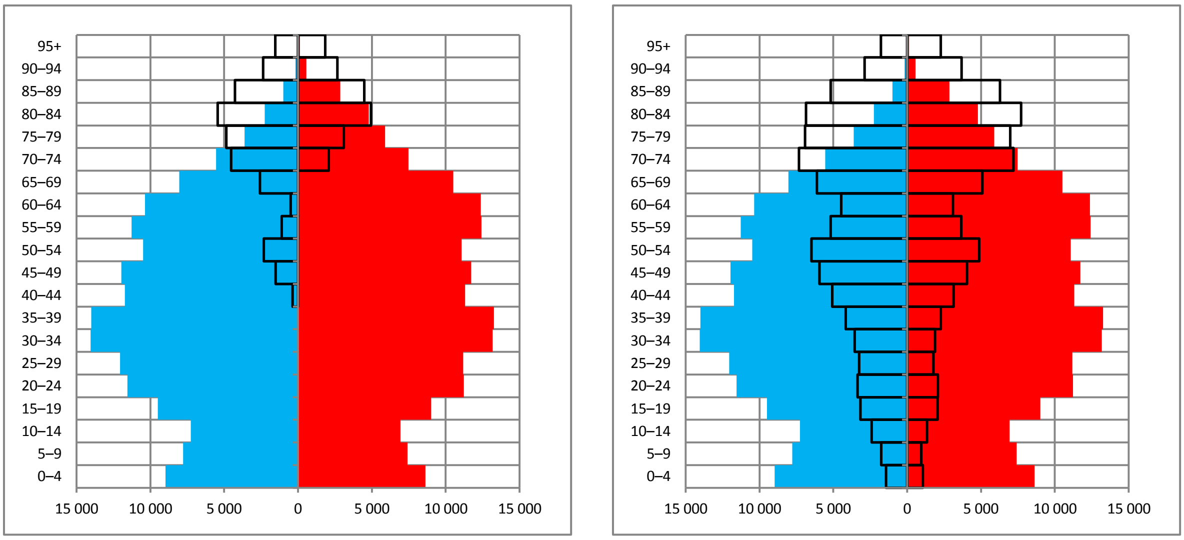

Fig 8: Demographic structure of district Ostrava at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

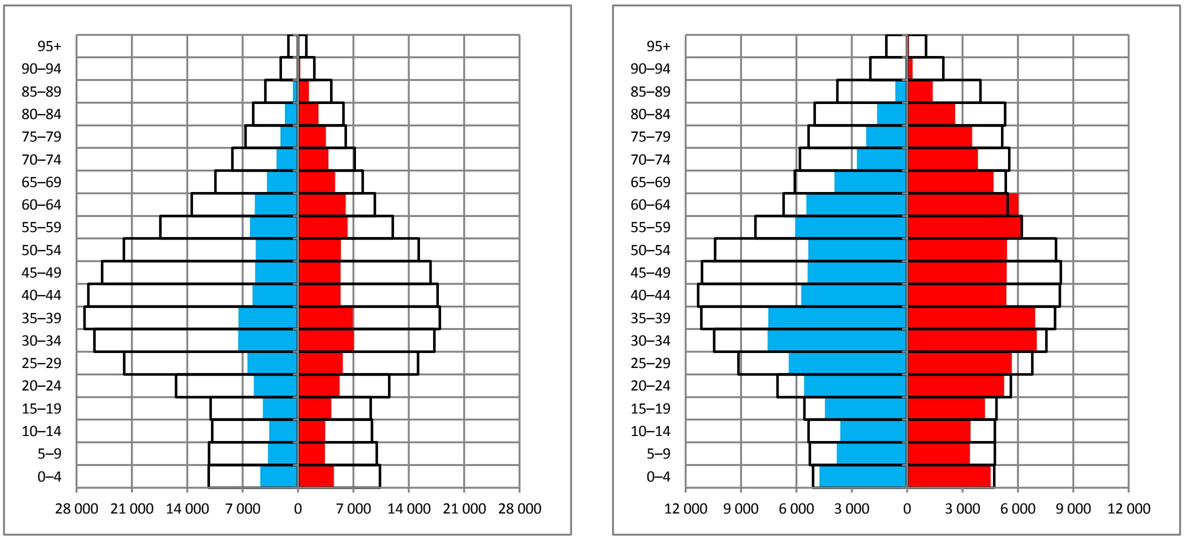

Fig 9: Demographic structure of district Pardubice at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

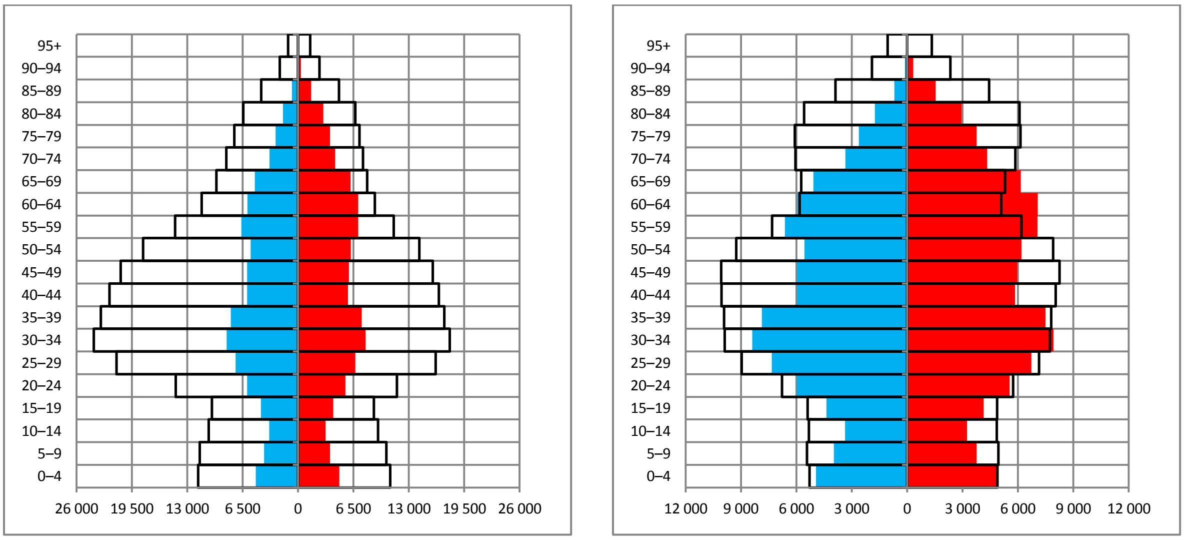

Fig 10: Demographic structure of district Plzeň at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

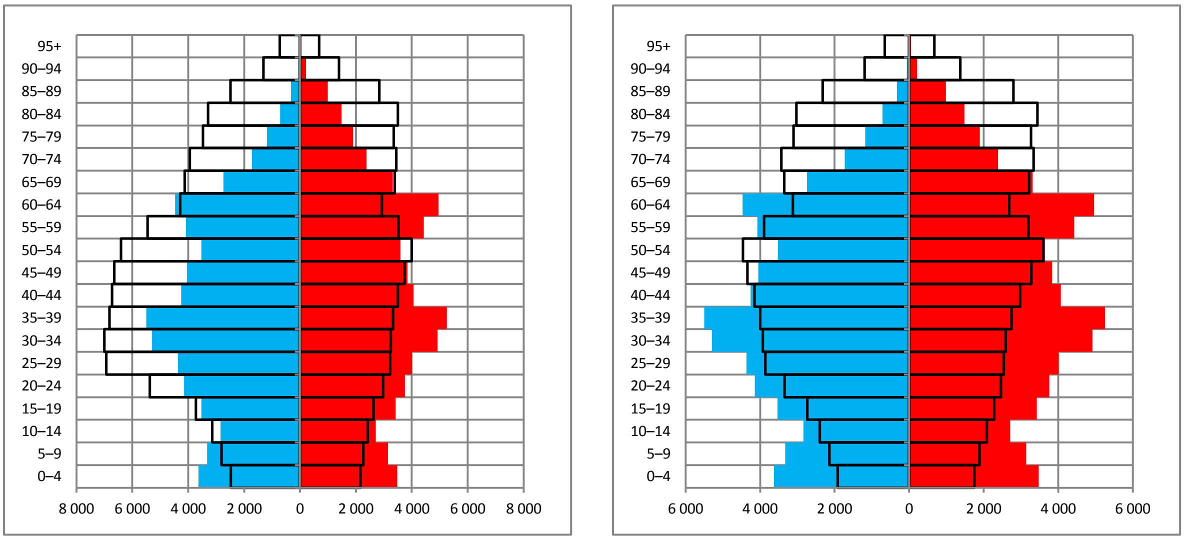

Fig 11: Demographic structure of district Ústí nad Labem at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

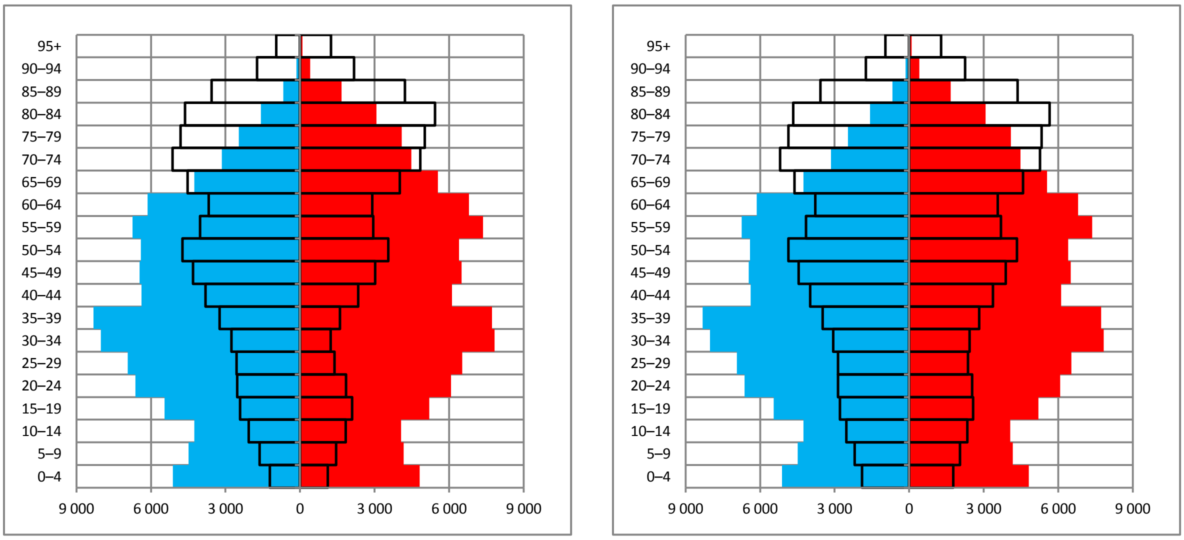

Fig 12: Demographic structure of district Zlín at 1.1.2011 and 1.1.2061 with constant trend of migration (left) and with exponential decay of migration (right) Source: CZSO, author's construction

CONCLUSION

In the case of the district Brno is not so noticeable the difference between constant trend of migration and exponential decay of the migration trend, because the migration growth in Brno is quite small. In the case of České Budějovice, Hradec Králové, Jihlava, Karlovy Vary, Liberec and Olomouc is more appropriate to consider the migration trend with exponential decay, especially because in some cases (see Liberec) would in horizon of 50 years occurred almost tripling of the population, which seems to be rather unlikely, with regard to spatial planning. (Population would have to live somewhere). Ostrava is experiencing in most recent years, practically the greatest depopulation ever. In the event that this depopulation trend will stay the same for all 50 years, practically almost no one will live there in the future. Therefore, we will assume that this strong depopulation once gets into the balance and exponentially falls. It is more likely. Districts Pardubice and Plzeň also have a very strong migration increases and thus will be more realistic to consider a model with exponential decay of the migration trend. In the case of Ústí nad Labem and Zlín are the results from both of two scenarios similar. Ustí nad Labem has significant increase in migration of men who, if they will stay here at the end of their productive life, grow old and the structure will stay so skewed.

REFERENCES

[1] BERGE, C. (1962), *„The Theory of Graphs and its applications“*, London : Methuen, New York : Wiley, 1962, 511 p.

[2] BOZICK, J. E., BELL, WILLIAM R. (1987), „Forecasting Age Specific Fertility Using Principal Components“, *Proceedings of the American Statistical Association, Social Statistics Section*, p 369–401, 1987.

[3] DOTLAČILOVÁ, P., LANGHAMROVÁ, J., ŠIMPACH, O. (2012), „Vybrané logistické modely používané pro vyrovnávání a extrapolaci křivky úmrtnosti a jejich aplikace na populace vybraných zemí Evropské unie“. *Forum Statisticum Slovacum*, 2012, roč. 8, č. 7, s. 21–25.

[4] FIALA, T. (2002), „*Demografické výpočty v tabulkovém procesoru*“. 1. vyd. Praha : Oeconomica, 2002. 218 s.

[5] FIALA, T., LANGHAMROVÁ, J. (2009), „Projekce obyvatelstva v ČR a jejích krajů.“ *Forum Statisticum Slovacum*, 2009, roč. 7, č. 5, s. 44–55.

[6] KALE, B., ROBERTSON, D.E., PALIT, Ch.D., VOSS, P. R. (2005), „The Migration Component in a Population Projections Model, *Proceedings of the Survey Research Methods Section*, American Statistical Association, USA, 2005, pp 1970–1976.

[7] KEYFITZ, N. (1964), „The population projection as a matrix operator", *Demography*, Springer-Verlag, Volume 1, Issue 1, March 1964, pp 56-73.

[8] KOSCHIN, F. (1993), „*Aktuárská demografie*". 1.vyd. Praha : Oeconomica VŠE, 1993. 141 s.

[9] LEE, R. (2000), "The Lee-Carter Method for Forecasting Mortality, with Various Extensions and Applications", *North American Actuarial Journal*, January 2000, Vol. 4, Iss. 1.

[10] PAVLÍK, Z., RYCHTAŘÍKOVÁ, J., ŠUBRTOVÁ, A.: „*Základy demografie*", 1. vyd. Praha : Academia, 1986. 732 s.

[11] PESARANA, H.H., SHIN, Y. (1998), „Generalized impulse response analysis in linear multivariate models", *Economics Letters*, Volume 58, Issue 1, 1 January 1998, Pages 17–29.

CONTACT TO THE AUTHOR

Ondřej, Šimpach, Ing.
W. Churchill sq. 4, 130 67 Prague 3, Czech Republic
+420 224 09 5273, +420 737 66 54 61
ondrej.simpach@vse.cz