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DOES TECHNICAL EFFICIENCY CALCULATED BY SFA STATISTICALLY SIGNIFICANTLY VARIES IN DIFFERENT SAMPLES?

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Abstract. The aim of the paper is to analyse whether and how the results of Stochastic Frontier Analysis (SFA) changes if we use different samples of firms. We do not question the size of the sample, but the composition of it. SFA is frequently used method to calculate the technical efficiency of agricultural holdings and authors come often to different results. The results of SFA are sometimes used for policy-making and decision-making in agriculture and the results must be reliable.
Agricultural holdings differ, and those differences can influence the results of SFA, i.e. the value of technical efficiency. We examined this issue empirically on a sample of 548 farms with 2268 observations for years 2013 to 2017. Accounting data were taken from Albertina database, data about acreage form LPIS. We chose randomly 5 samples per 500 observations (the number of farms differed as the panel was unbalanced).
Technical efficiency for each sample was calculated by the same technique. Particularly True Fixed Effects model was used to model the Cobb-Douglas production function. Technical efficiency was calculated by Jondrow et al. (1982) method. Differences in medians of technical efficiency were tested by non-parametric Kruskal-Wallis test and Mood's median test. H0: there are no statistically significant differences in medians of technical efficiency was rejected in the first test but was not rejected in the other. We came to ambiguous results but nevertheless we can warn that the selection of the sample for calculation of technical efficiency might influence the results.

Keywords: technical efficiency, hypothesis testing, stochastic frontier analysis

Mathematics Subject Classification: Primary 91G70; Secondary 62P20

1 Introduction

Agricultural holdings are very heterogenous due to many reasons. One of them are climate differences, different altitude, various type of management (family versus corporates) or production focus (mixed, animal, crop production). Hence, calculation of technical efficiency may be influenced by the composition and construction of the sample. Especially non-parametric Data Envelopment Analysis (DEA) is sensitive method to the outliers. For example, Johnson and McGinnis [8] proposed the method that will help to identify outliers and thereby improve the accuracy of second stage results in two-stage nonparametric analysis. In parametric Stochastic Frontier Analysis (SFA), the problem of outliers is not that important, but still the possible differences in technical efficiency shall be observed as SFA is widely used and the results are often utilized in decision and policy-making.

Pechrová and Vlašicová [19] or Pechrová [16] examined the effect of the agricultural subsidies on technical efficiency and found negative effect. In food processing industry Naglová and Šimpachová Pechrová [13] also found out that the increase of subsidies caused a slight increase in the mean of technical inefficiency. Similarly, Latruffe et al. [11] found that subsidies had negative impact on technical efficiency of farms in the Czech Republic in years 2000–2004. Technical efficiency of different types of agricultural holdings was examined also by Čechura [2] by Fixed Management model for calculation of technical efficiency and total factor productivity of the total agriculture and its individual branches. He concluded that "technical inefficiency is an important phenomenon in Czech agriculture, i.e. the efficiency differences among companies are an important reason for the variations in production; this holds true for both intersectoral and intrasectoral comparisons" Čechura [2]. Čechura and Matulová [3] analysed using SFA the changes in technology and technical efficiency of sectors of Czech agriculture after the EU enlargement in 2004. However, they found none statistically significant inter-sectoral differences in technical change. Pechrová and Šimpach [18] assessed whether there are differences between agricultural holdings managed by males or females, but despite that they were differences (females seemed more technically efficient), they were not statistically significant. Differences between young and other farmers in technical efficiency were examined by Pechrová [17], but there were found none.

Because of its wide utilization, it is necessary to examine the SFA whether the sample matters and influence the value of technical efficiency. Hypothesis about equality of multiple independent means is tested by ANOVA. When at least one mean statistically significantly different it is usually followed by Tuckey test to examine which means differ. Non-parametric testing hypotheses in two-sample problem concerned e.g. Nishino and Murakami [14]. Multiple non-parametric testing is presented and empirically performed in the paper of Beuren and Anzanello [1]. However, this test assumes normal distribution of the tested variable. Technical efficiency takes values from the interval between 0 to 1 and is usually skewed to the left. SFA compares the firms among each other and those lying on the efficiency frontier are 100% efficient, others lay under it, many of them close to the frontier. Therefore, non-parametric tests must be used (see e.g. Grech and Calleja [6]). To evaluate whether two or more samples are drawn from the same distribution is used Kruskal–Wallis test (Guo, Zhong and Zhang [7]) or Mood's median test (Chen [4]). Both tests are introduced in the next section. Then the results of the SFA and calculated technical efficiency are presented. Next section discusses the results. Last section concludes.

2 Data and Methods

The aim of the paper is to analyse whether and how the results of SFA changes different samples of firms are used. A parametric method – Stochastic frontier analysis (SFA) was used to calculate technical efficiency of several samples of agricultural holdings. The methods evaluate the efficiency of each firm in comparison to the others. Hence, the efficiency changes, when the sample changes. We examined this empirically on a sample of 548 farms with 2268 observations for years 2013–2017 (unbalanced panel). Accounting data were taken from Albertina database, data about acreage from LPIS. We chose randomly 5 samples per 500 observations (the number of farms differed as the panel was unbalanced). Their description is at Table 1. Explained variable – production – was represented by sales of own products. Its range was between 47 mil. CZK to 64 thous. CZK. Explanatory variables were production factors: material (usage of material and energy) in thous. CZK ranging from 23 to 32 mil. CZK, tangible assets in thous. CZK in the interval from 62 to 93 mil. CZK, number of employees from 29 to 35 and acreage of land (in hectares) from 860 to 1212 ha.

Variable	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
$y_{, it}$ – production (thous. CZK)	47080	64632	55141	51507	60647
$x_{1, it}$ – material (thous. CZK)	23312	32139	27332	26134	24807
$x_{2, it}$ – tangible assets (thous. CZK)	62319	93311	61596	68484	67207
$x_{3, it}$ - number of employees	29	35	33	30	31
$x_{4, it}$ - acreage of land	940	1212	860	892	839

Tab. 1. Arithmetic mean of the samples of agricultural holdings
Source: Albertina database, LPIS, own elaboration

To eliminate the influence of time, the value of production was adjusted by the index of agricultural producers. Prices of material and tangible assets were also adjusted by index of industrial producers. Technical efficiency for each sample was calculated by the same technique. Particularly True Fixed Effects model as defined by Greene [5] was used to model the Cobb-Douglas production function in the form (1) linearized by natural logarithms (2).

$$y_{it} = x_{1,it}^{\beta_1} x_{2,it}^{\beta_2} \dots x_{4,it}^{\beta_5} e^{u_{it}} e^{v_{it}} \tag{1}$$

$$y_{it} = \beta_1 \ln x_{1,it} + \beta_2 \ln x_{2,it} + \dots + \beta_4 \ln x_{4,it} + u_{it} - v_{it} \tag{2}$$

where y_{it} is explained variable (production the i^{th} firm at time t), $\beta_1 \dots \beta_5$ are coefficients of explanatory variables x_{it} (production factors), u_{it} represents technical inefficiency of particular agricultural holding i in time t ($u_{it} \geq 0$) and v_{it} is pure stochastic noise (idiosyncratic error component). Cobb-Douglas (power) function has the advantage that the coefficients can be interpreted as a percentage change of the variables and the sum of the coefficients express the type of returns to scale (< 1 – decreasing, = 1 – constant, > 1 – increasing). The distribution of the inefficiency term u_{it} was assumed to be exponential, or half-normal or truncated normal according to the situation which model fitted the best. Stochastic term v_{it} is assumed to be normally distributed. After estimation of the production function, the technical efficiency was calculated by Jondrow et al. [9] method via exp[-E(u|e)]. The differences in technical efficiency among samples was examined next.

Whether there are any significant differences amongst the means is standardly tested by F-test in the ANOVA table with following Tuckey test if there are any that tells which means are significantly different from which others. However, these tests assume normal distribution of the variables. Technical efficiency takes values from 0 to 1 and is not normally distributed. It was checked by the calculation of skewness and kurtosis of the calculated variables. Hence, we choose the Kruskal-Wallis test which compares medians instead of means. H_0 assumes that there are no statistically significant differences in means of technical efficiency – the medians within each of the 5 samples is the same. It compares “two or more classes in relation to each variable *q* in order to test whether such classes originate from the same distribution based on a median rank” (Beuren and Anzanello [1]). The data from all the samples is first pooled and ranked from smallest to largest regardless to which of *K* samples they belong to. Then the rank is then computed for the data in each sample, so that each sample has its sum of ranks. If there is no tie in all the values, the test statistic is (3):

$$H = \frac{12}{N(N+1)} \sum_{k=1}^{K} \frac{R_i^2}{n_i} - 3(N+1) \tag{3}$$

where N is the total number of values in all samples; n_k is the number of values contained in the k^{th} sample, and R_i is the sum of ranks in k^{th} sample. (Guo, Zhong and Zhang [7]). Test statistics H is compared to the value of χ^2 distribution with K - 1 degrees of freedom. When H exceeds the critical value, null hypothesis is rejected. When the p-value is less than 0.05, there is a statistically significant difference at the 95% confidence level. To graphically determine which medians are significantly different from which others is used Box-and-whisker plot.

An alternative way, how to examine the medians of multiple samples is to use Mood's median test and following median plot. It tests the hypothesis that the medians of all 5 samples are equal. It does so by counting the number of observations in each sample on either side of the grand median, i.e. median calculated from the samples pooled in one “grand” sample. Mood’s test statistic is defined as follows (4):

$$M = 4\sum_{k=1}^{K} \frac{\left(A_k - n_k / 2\right)^2}{n_k} \tag{4}$$

where K is the number of groups, each having n_k (k = 1, 2, ..., K) observations, A_k is the number of times that the ranks of individual observations from group k excess the overall median from the pooled data. “Asymptotically, under the null hypothesis that all samples have the same median, the test statistic in (4) has a χ^2 distribution with K - 1 degrees of freedom.” (Chen [4]). Test statistics is compared with critical value of χ^2 distribution or p-value can be also calculated. If the p-value is lower than 0.05, there is a statistically significant difference amongst the medians at the 95% confidence level. The medians can be further examined in median plot which displays also 95% confidence intervals. Calculation of SFA was done in Stata 15.1 and the tests were performed in Statgraphics Centurion XVI.

3 Results

First, the SFA method was used to estimate Cobb-Douglas production function in each of 5 samples. The aim was to use the same model, but with different distribution of an inefficiency term which would fit the best the data. Each coefficient had to be statistically significant at least on 0.1 level of significance. This was checked by t-test. Also, the model as a whole had to be statistically significant according to Wald χ^2 test. When predicting efficiency, some observations had to be dropped from the analysis. Hence, the numbers are not equal 500, but less (see Table 2). The highest technical efficiency was estimated in sample 4 – 86.55% and the lowest in sample 5 – 77.38%, but the highest median was in sample 3 – 91.90% and the lowest again in sample 4 – 88.33%. Values in other samples were more or less the same. It seems on the first sigh that sample 5 might be different and its value may differ from the other samples. There is also the highest standard deviation which points out on higher variability in the data. The standardized skewness and/or kurtosis is outside the range of -2 to +2 in all 5 samples. This indicates some significant nonnormality in the data.

Sample	Distribution of u_{it}	Technical efficiency						
		Nr.	Mean	Median	Std. dev.	Min.	Skewness	Kurtosis
1	exponential	485	0.8404	0.9118	0.2050	0.0122	-16.2269	12.8593
2	half-normal	481	0.8382	0.9022	0.2131	0.0307	-18.0169	17.1914
3	exponential	472	0.8528	0.9190	0.2029	0.0211	-19.5047	21.6700
4	exponential	481	0.8655	0.9038	0.1681	0.0449	-19.3732	25.2744
5	truncated normal	481	0.7738	0.8833	0.2753	0.0005	-11.5796	29.6090
Total		2400	0.8341	0.9049	0.2179	0.0005	-37.8106	32.3247

Tab. 2. Descriptive statistics of the technical efficiency in 5 samples of agricultural holdings
Source: own elaboration

Kruskal-Wallis test tests the null hypothesis that the medians within each of the 5 samples is the same. Results of Kruskal-Wallis test are displayed in Table 3. χ^2 test with 4 degrees of freedom (n-1) shows that the probability that the H_0 holds is only 0.0001 (p-value). There is at least one difference between medians at the 95.0% confidence level. Which medians are different is a subject of further graphical analysis in Box-and-Whisker Plot.

Sample	Number	Average rank	Rank sum	Test
1	485	1260.76	611467.50	χ^2 = 47.0150 with 4 d.f.
2	481	1165.42	560567.50	p-value = 0.0001
3	472	1252.57	591215.00	
4	481	1297.09	623898.50	χ^2 with ties = 47.095 with 4 d.f.
5	481	1027.13	494051.50	p-value = 0.0001

Tab. 3. Results of Kruskal-Wallis equality-of-populations rank test
Source: own elaboration

Box-and-whisker plot (see Fig 1.) summarizes data samples through 5 statistics: 1. minimum (dot on the left side) 2. lower quartile (left side of the “box”) 3. median (i.e. second quartile – line inside of the “box”) 4. upper quartile (right side of the “box”; in our case it is always 1), 5. maximum (in our case it is always 1). The dot inside the “box” represents an arithmetic mean. Outside points, which are points more than 1.5 times the interquartile range (box width) above or below the box, are indicated by point symbols. In our case, all points are on the left side under the “whisker” that indicates less than 1.5 times the interquartile range (higher quartile minus lower quartile). There are outliers. There are many of them as the sample is quite large. Median in the fifth sample is different than the others.

Alternative way of testing the differences is by Mood's median test with same H_0 that all medians of 5 samples are equal. The results are displayed at Table 4. Grand median equals 0.9049 and observations in the samples are compared to it. The number of values that are below or above the grand median is counted and test statistics is calculated (6.3599).

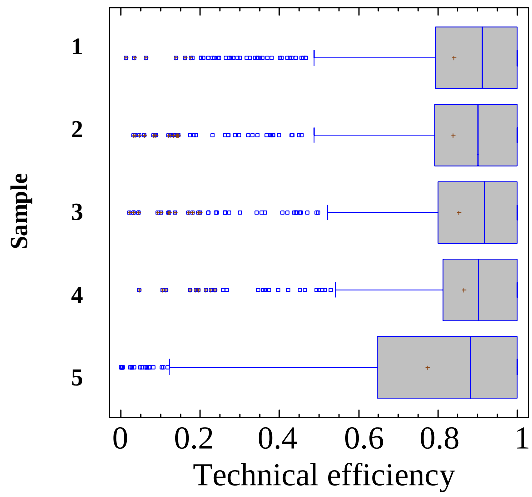

Fig. 1. Box-and-whisker plot of technical efficiency
Source: own elaboration in Statgraphics Centurion

It follows χ^2 distribution. P-value for the χ^2 test is greater than 0.05, the medians of the samples are not significantly different at the 95,0% confidence level. Table 4 also displays 95.0% confidence intervals for each median based on the order statistics of each sample.

Sample	Number	n <=	n >	Median	95.0% lower CL	95.0% upper CL	Test
1	485	239	246	0.9118	0.8923	0.9262	Test statistic M = 6.3599
2	481	242	239	0.9022	0.8851	0.9183	
3	472	217	255	0.9190	0.9023	0.9323	
4	481	242	239	0.9038	0.8844	0.9263	p-value = 0.1738
5	481	260	221	0.8834	0.8551	0.9087	

Tab. 4. Results of Mood's median test
Source: own elaboration

Medians and confidence intervals can be displayed in median plot – see Fig 2. The medians are not statistically significantly different as their confidence intervals are overlapping.

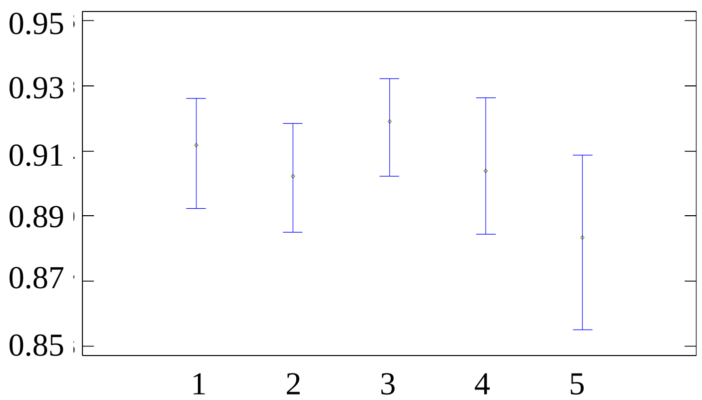

Fig. 2. Median plot with 95.0% confidence intervals of technical efficiency
Source: own elaboration

4 Discussion

We came to ambiguous results. Kruskal-Wallis test revealed that medians of the technical efficiency in 5 samples are not equal, but Mood's median test indicates the opposite. First test's plot suggests that median of the sample 5 might be different. On the other hand, drawing confidence intervals in median plot supports the results of Mood's median test. We cannot clearly conclude whether the technical efficiency of the sample differs. When we want to estimate the technical efficiency of a sample of agricultural companies, we must pay attention to the size and composition of the sample. The value of technical efficiency might be influence by the content of the sample. The sample therefore shall be as large as possible to obtain accurate value of technical efficiency. Kruskal–Wallis test is often used on small sized data (Guo, Zhong and Zhang [7]). In large samples, where *n* (size of the sample) is close to *N* (size of the population), the probability of rejection of null hypothesis is approaching to 1.0. P-value is close to one. On the other hand, SFA shall be applied on large sample to increase the probability that the estimated parameters of the model are statistically significant.

We think that there might be differences as we observe for example results of research that considers organic farms. There are about 4696 organic agricultural holdings in the Czech Republic nowadays (MoA [12]). The number of farms has increased rapidly between 2004 (after the entrance to the European Union) and 2010. Since that, the increase of the organic agricultural holdings is not that fast and there is a high probability that when there is a research on technical efficiency done by different authors, their sample would contain same farms. Despite that and that the number of organic farms is relatively low (in comparison the number of all agricultural holdings in the Czech Republic), the results differ a lot.

Pechrová and Vlašicová [19] estimated based on panel data for period 2005–2012 the technical inefficiency and efficiency of biodynamic and organic farms in the Czech Republic and compare them with each other. They found statistically significant differences in the inefficiency and efficiency of resources usage between biodynamic (average efficiency 65.9%) and organic (79.1%) farms. Kroupová, [10] examined the differences between conventional and organic farms and found that organic farms have lower technical efficiency by 13.5%, mainly due to the fact that they are located in less favourable areas (70% of the sample). The average efficiency of organic farms according to Kroupová [10] was only 55.1% in 2004–2008, while according to Pechrová and Vlašicová [19] it was even 79.1% in 2005–2012. Pechrová [15] found that average organic farm produced 79.4% of the potential product with the standard deviation of 19.4% and only 35.6% of the observations were in less favourable areas. There was similar sample (from the same period) as in case of the article Pechrová and Vlašicová [19], but the type of model was different. So, the results are almost the same. But the difference from the Kroupová's [10] research is significant. Of course, the time period was different, but it is probable that the sample of Pechrová [15] and Pechrová and Vlašicová [19] included at least some same farms. The variability and differences among organic farms influenced the results too. We therefore can conclude that when doing research about agricultural holdings, the composition of the sample. We could not test the differences on more samples, as the technical efficiency has to be calculated individually for each sample of agricultural holdings (and an optimal model has to be found for each production function) and this process cannot be automated. Challenge for future research is to enlarge the number of samples and calculate more values of technical efficiency.

5 Conclusion

We analysed if the results of Stochastic Frontier Analysis (i.e. value of technical efficiency) changes if we use different samples of agricultural holding. SFA is frequently used method to calculate the technical efficiency of agricultural holdings and authors come often to different results. The results of SFA are sometimes used for policy-making and decision-making in agriculture and the results must be reliable. Agricultural holdings differ, and those differences can influence the value of technical efficiency. Based on 5 samples per 500 observations randomly selected from 2268 observations for years 2013 to 2017. Technical efficiency for each sample was calculated by the same technique. Particularly True Fixed Effects model was used to model the Cobb-Douglas production function. Technical efficiency was calculated by Jondrow et al. [9] method. Average efficiency of the sample 1 was 84.04%, sample 2 83.82%, sample 3 85.28% and sample 4 86.55%. Efficiency calculated in sample 5 was the lowest – only 77.38%. We tested the differences in medians of technical efficiency by non-parametric Kruskal-Wallis test and Mood’s median test. The first mentioned revealed that at least one median is statistically different from the others. According to the box-and-whiskers plot the median in sample 5 was different from the others. On the other hand, Mood’s median test considered all medians as equal. We came to ambiguous results but nevertheless we can warn that the selection of the sample for calculation of technical efficiency might influence the results, because we showed on organic farms that results of different researches differ.

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