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POLYNOMIAL FUNCTIONS AND SMOOTHING OF MORTALITY RATES: THE CZECH REPUBLIC AND SLOVAKIA DURING THEIR INDEPENDENT DEVELOPMENT AFTER THEIR SEPARATION

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Abstract. In this paper we present the possibility of use of polynomial functions of the 2nd and 3rd order for modelling of mortality in the case of Czech Republic and Slovakia during their independent development after separation in 1993. Mortality significantly affects the length of human life and the approach which is used in this paper clearly show differences between mortality of the Czech and Slovak population. Suitability of used methods is tested by statistical t-tests and at the end we discuss about the circumstances in mortality differences between compared populations.

Key words: Polynomial function, mortality, age-specific death rate, Czech Republic, Slovakia.

Mathematics Subject Classification: Primary 90C30; Secondary 62H12.

1 Introduction

Mortality at the advanced ages becomes more and more important topic for demographers and for analysts in the field of health and pension insurance (Fiala [9]). Extensions of length of life is influenced mainly by improvement in medicine and in healthcare. At first, there was a significant improvement in a care of live born persons and infants, which caused the decrease of infant mortality and mortality rates during childhood. Later, mortality began to improve even at the advanced ages (Dotlačilová, Šimpach [4]). Among reasons for this evolution could be included higher level of health care, standard of living, better living conditions or functional health and pension system. Another equally important reason could be more interest in a healthy lifestyle (Šimpach, Langhamrová [14] or [15]) and also better environment in most countries (especially in large industrial cities). Given to this evolution, it is more and more important to have the best imagination about how long in average will live not only the youngest persons, but also the oldest ones. From the past analysis, performed e.g. by Boleslawski, Tabeau [1] or Burcin, Tesárková, Šídlo [3] it is evident that the level of mortality of younger persons is different in comparison with the oldest ones (Gavrilov, Gavrilova [10]). Therefore, it is necessary to correct estimates of mortality at the highest ages. For this correction are used various types of models. Given to the importance of the most accurate capture of mortality at the advanced ages are still required new models and methods that would provide the best imagination of current trends (see Fiala [8] or Dotlačilová, Šimpach, Langhamrová [5], [6]). Nowadays we can use several existing models which are used for smoothing and for estimating of unknown parameters. It is possible to obtain them by using the professional demographical software (e.g. DeRaS, see Burcin, Hulíková Tesárková, Kománek [2]). Among the most famous are included Coale-Kisker model (see e.g. Boleslawski, Tabeau [1] or Gavrilov, Gavrilova [10])

$$m_{x,t} = e^{ax^2+bx+c}, \tag{1}$$

where $m_{x,t}$ are age-specific mortality rates and x is age. Obtained model corresponds with an exponential quadratic function, where a, b and c are parameters of model. Next one is Thatcher model (see e.g. Thatcher, Kanistö, Vaupel [17])

$$\mu_{x,t} = \frac{z}{1+z} + \gamma, \tag{2}$$

where $z = \alpha e^{\beta x}$, α, β, γ are parameters of model, x is age (and μ_x is the intensity of mortality at exact age x). The other one is Kannistö model (Thatcher, Kanistö, Vaupel [17])

$$\mu_{x,t} = \frac{e^{[\theta_0+\theta_1(x-80)]}}{1+e^{[\theta_0+\theta_1(x-80)]}}, \tag{3}$$

where θ_0 and θ_1 are unknown parameters. Kanisto model is a special case of the logistic function, where logit transformation of mortality rates is expressed by linear function of age. The oldest model (but still very frequently used) is the Gompertz-Makeham function (Gompertz [11], Makeham [13])

$$\mu_{x,t} = a + bc^x, \tag{4}$$

where a, b and c are parameters (more e.g. in Ekonomov, Yarigin [7], or Šimpach [16]). All these models are suitable for the elimination of fluctuations in age-specific death rates and their subsequent extrapolation. Another option is application of polynomial functions.

In this paper we use the polynomial of 2nd and 3rd order and estimation of parameters of regression functions will be based on the age range x = 60–85 years. We evaluate the statistical significance of the parameters by individual t-tests, and the parameter's significance will be one of the most important information about the suitability of model. The other information about suitability of model will be comparison of empirical age-specific mortality rates with smoothed and extrapolated values in a graphical form prepared by 3D Bicubic spline in Statgraphics Centurion XVI.

2 Materials and Methods

For the purposes of mortality analysis we use demographic data from the "Human Mortality Database" (HMD [12]) for the Czech Republic and Slovakia, with annual frequency of observations. Let us denote the polynomial function of 2nd and 3rd order as

$$m_{x,t} = \begin{cases} \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \varepsilon_t \\ \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \beta_3 x_t^3 + \varepsilon_t \end{cases}, \quad (5)$$

where $m_{x,t}$ are the age-specific mortality rates calculated by ratio

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}}, \quad (6)$$

where $D_{x,t}$ is the number of died x-years old persons in calendar year t, $E_{x,t}$ is the exposure to risk of x-years old persons in calendar year t, which is (commonly) estimated as a number of mid-year population x-years old in year t, β_0, β_1, β_2 and β_3 are parameters of polynomial functions and ε_t is the error term with characteristics of white noise, x is age where $x \in <0\,;\,110>$, (the estimation of parameters of polynomial functions will be based on the age range x = 60–85 years only), and t is time where $t \in <1993\,;\,2011>$ for the case of the Czech Republic and $t \in <1993\,;\,2009>$ for Slovakia. Slovak time series unfortunately end in 2009 and recent estimates by the HMD have not been published yet. These polynomial functions we use for smoothing of age-specific mortality rates from 60 to 85 years. This interval is not obligatory, but there are some literature references (e.g. Burcin, Tesárková, Šídlo [3]), which recommend it. For the advanced ages we perform the extrapolation and our recommendation for the end is x = 110.

In some years is the highest completed age at the appropriate population according to HMD less than x = 110 years. It means that data is missing, or the data matrix is inaccurate, or there was not any person in a given year, who achieved this advanced age. In this case for the purpose of displaying charts of empirical age-specific mortality rates we impute missing values as

$$m_{x+1,t} = m_{x,t} \times 1{,}01, \quad (7)$$

where x+1 is the first unknown observation in a given year t, because we assume the law of mortality the same as expected B. Gompertz (Gompertz [11]) and later W. Makeham in his actuarial calculations (Makeham [13]).

3 Results

We show in the Fig. 1 the evolution of empirical (observed) age-specific mortality rates for males and for females in the Czech Republic. We focus only on their development between 60 and 110 years of human life, because the smoothing using polynomial functions is performed between the ages 60 and 85 years. For ages 85 and more values are obtained by extrapolation according to an estimate of polynomial model. From the obtained results there is already evident, why it is good to smooth the empirical values – because the age-specific mortality rates at the highest ages do not show any clear trend (this contention is valid approximately from 85–90 years).

In the Fig. 2 is shown smoothing of empirical mortality rates using the polynomial function of the 2nd order (and the subsequent extrapolation of these values up to 110 years). Here it is evident that in the case of the highest ages there were eliminated major fluctuations, which were clear from the graphical output for empirical values. When we compare the obtained results with the empirical rates, we find that in the age range 60–80 years we receive for both gender very good smoothing. But in the case of higher ages there become undervaluing of the observed rates.

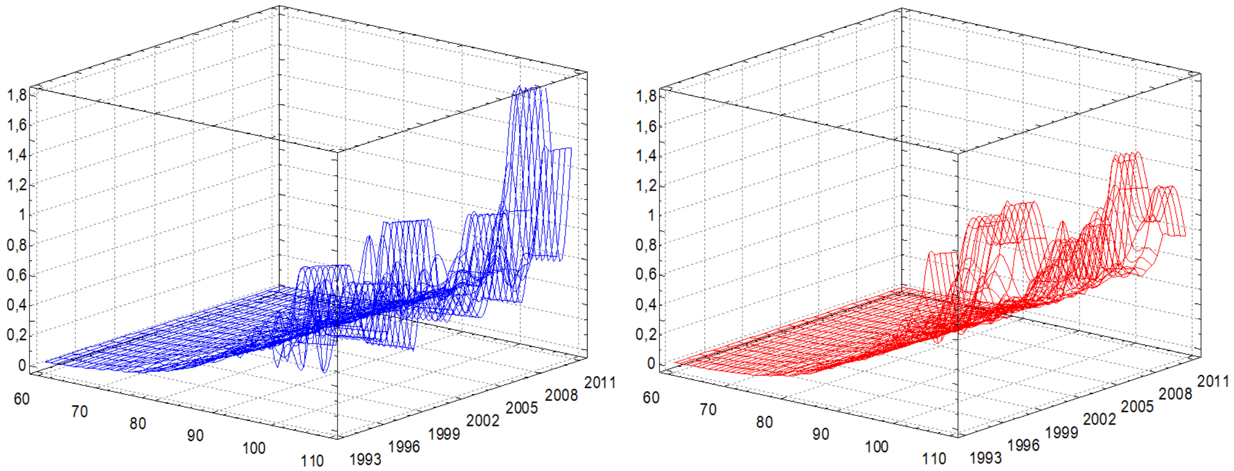

Fig. 1. Empirical values of age-specific mortality rates of Czech males (left) and females (right).
Source: Human mortality database, authors’ construction.

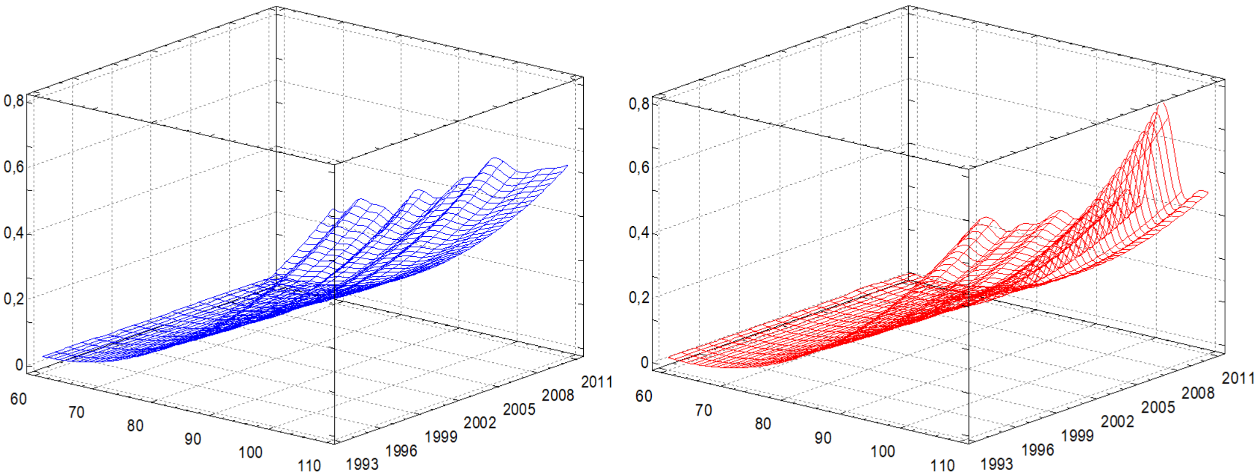

Fig. 2. Smoothing by polynomial function of the 2nd order for Czech males (left) and females (right).
Source: authors’ calculations and construction.

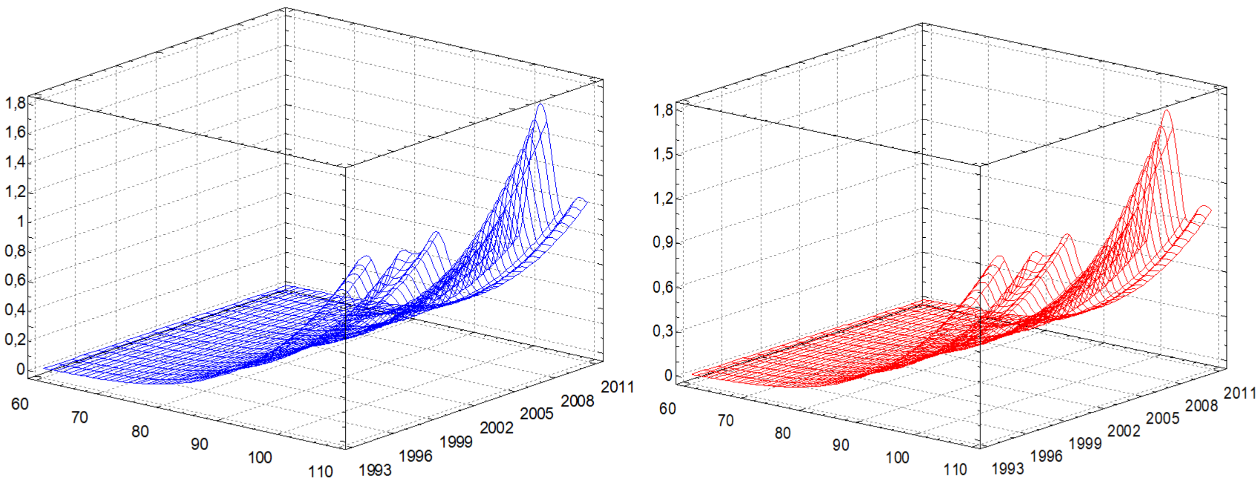

Fig. 3. Smoothing by polynomial function of the 3rd order for Czech males (left) and females (right).
Source: authors’ calculations and construction.

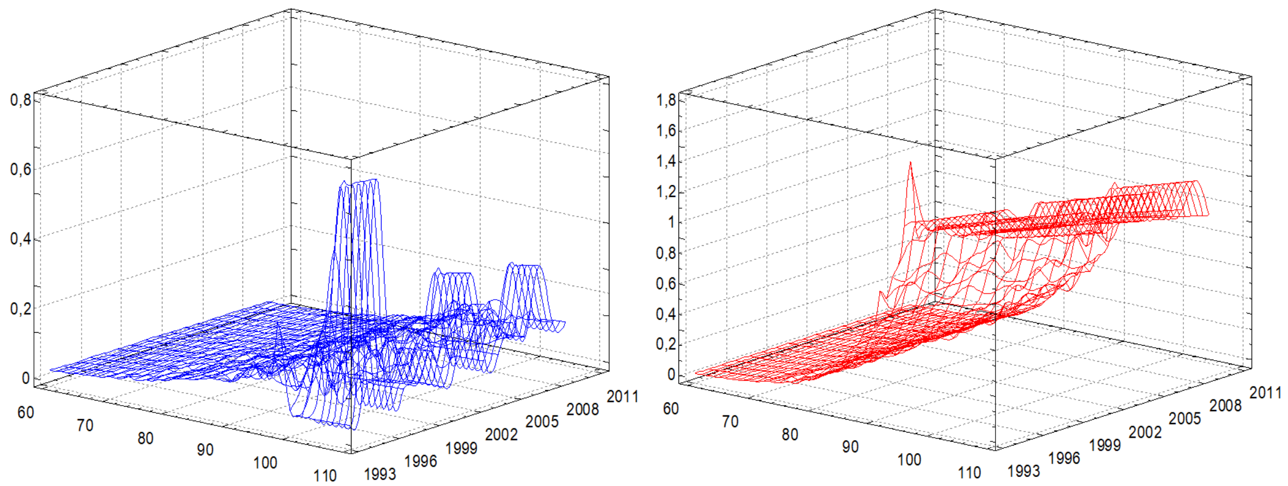

Fig. 4. Empirical values of age-specific mortality rates of Slovak males (left) and females (right).
Source: Human mortality database, authors' construction.

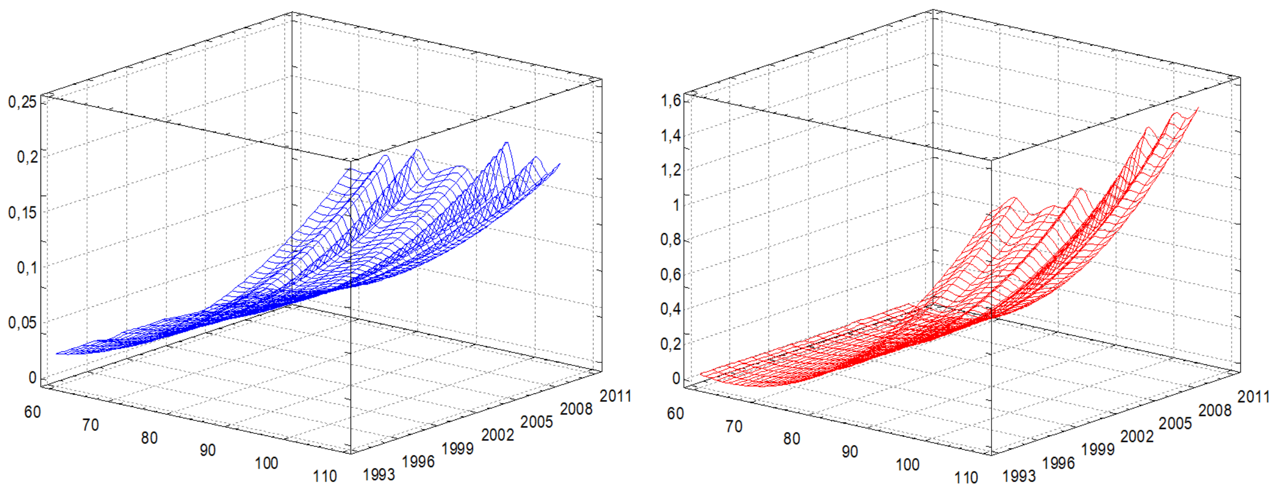

Fig. 5. Smoothing by polynomial function of the 2^{nd} order for Slovak males (left) and females (right).
Source: authors' calculations and construction.

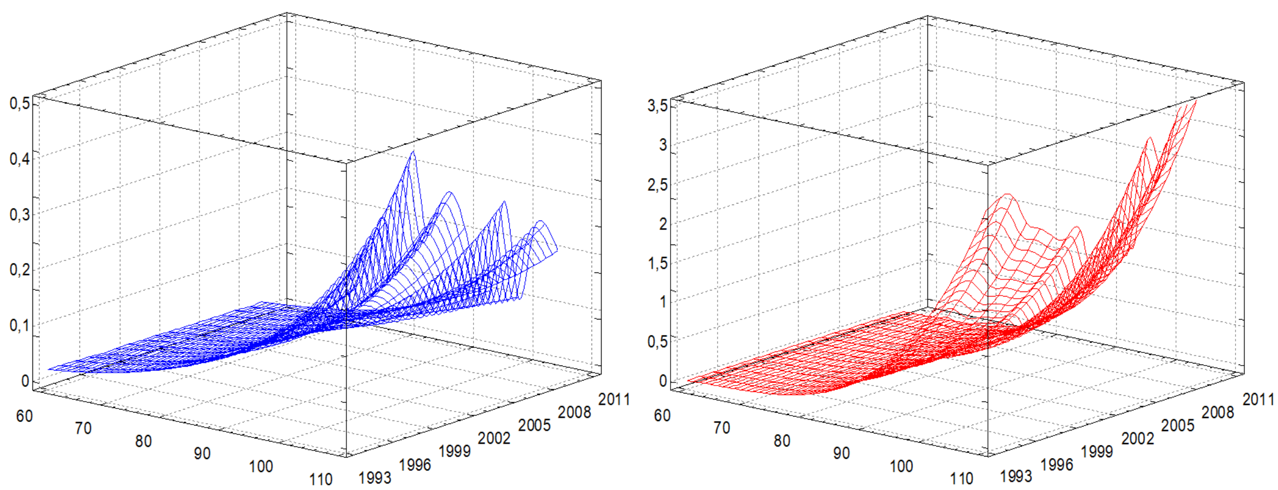

Fig. 6. Smoothing by polynomial function of the 3^{rd} order for Slovak males (left) and females (right).
Source: authors' calculations and construction.

In the Fig. 3 is shown smoothing using the polynomial function of the 3rd order. Same as the case of polynomial function of the 2nd order also there have been clearly smoothed mentioned fluctuations. But if we compare smoothing by these two types of polynomial functions, we find out that the polynomial function of the 3rd order provided slightly different results. If we focus our attention on lower age range (approximately from 60 to 80 years), we find that there is very good levelling (in the comparison with empirical values). In the case of higher ages (approximately for 80 years and more) there become the undervaluing of observed rates.

The next group of charts shows the actual age-specific mortality rates in the Slovak population of males and females in the age range of 60–110 years for the period 1993–2010. Even here it is evident that the highest age (roughly 90 years for males and 85 years for females experiencing significant fluctuations). Also here (in the case of empirical data) (see Fig. 4) is evident, that at the highest age (about 90 years for males and 85 years for females) there are significant fluctuations. The most significant are changes that primarily occurred around 2000. From obtained results is also clear the different development of mortality of Slovak males and females.

In the fifth group of charts (see Fig. 5) is shown smoothing of Slovak mortality rates by the polynomial functions of the 2nd order. If we compare results obtained with the empirical data (see Fig. 4), we can conclude that mainly in the age range from 60 to 80 years we obtain relatively good smoothing. On the other hand, from results at higher ages is evident that the 2nd order of polynomial function provides lower level of mortality (and therefore it underestimates observed values).

We can see the last group of charts (see Fig. 6) which shows smoothing of Slovak age-specific mortality rates (and subsequent extrapolation) using polynomial function of the 3rd order. When we closer examine values obtained in age range from 60 to 80 years, we conclude that here the polynomial function of the 3rd order provides a fairly good smoothing. For higher ages (especially 80 years and more) we could see, that mainly in the case of Slovak males we receive smoothing, which is more consistent with the actual level of mortality rates. For female's population it is unfortunately not so clear. Due to the large fluctuations in empirical values of age-specific mortality rates, we get balanced values that significantly overestimate their original values (approximately from 2005) when we apply a polynomial function of the 3rd order.

4 Discussion and Conclusion

Based on the obtained results we could say that both (polynomial function of the 2nd order and polynomial function of the 3rd order) provides quite good smoothing in the age range from 60 to 80 years. For ages over 80 years we unfortunately cannot make a clear conclusion. If we focus our attention on mortality of the Czech population, we can conclude that for the highest ages it is better to use the 3rd order of polynomial function. The polynomial function of the 2nd order provides for ages 80 years and over much more undervalued values of mortality rates (compared with the initial empirical observations of these rates). Based on the smoothed results we could also say that none of these functions is universally applicable for the whole age range (i.e. from 60 to 110 years).

Thus there arise the possible way of future research: combination of these two types of functions. From the obtained smoothed values there are particularly the most interesting age-specific mortality rates of Slovak females. When we use the polynomial function of the 2nd and the 3rd order, we do not get just perfect smoothing (especially for ages above 80 years). Given that mortality rates of Slovak females show significant fluctuations at the highest ages (see Fig. 4), it would be probably better to use some of more suitable model, which we talk about in the introduction part of our paper. It is Coale-Kisker model (1), Thatcher model (2), Kanistö model (3) and Gompertz-Makeham function (4).

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Appendix

	1993		1994		1995		1996		1997		1998		…	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	…	
β_0	1,006E+0	0,000	9,660E-1	0,000	1,022E+0	0,000	1,013E+0	0,000	9,393E-1	0,000	8,332E-1	0,000	…	
β_1	-3,232E-2	0,000	-3,109E-2	0,000	-3,278E-2	0,000	-3,221E-2	0,000	-3,008E-2	0,000	-2,707E-2	0,000	…	
β_2	2,668E-4	0,000	2,571E-4	0,000	2,697E-4	0,000	2,629E-4	0,000	2,475E-4	0,000	2,259E-4	0,000	…	
	1999		2000		2001		2002		2003		2004		…	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	…	
β_0	1,004E+0	0,000	1,008E+0	0,000	9,295E-1	0,000	9,939E-1	0,000	1,027E+0	0,000	1,059E+0	0,000	…	
β_1	-3,191E-2	0,000	-3,198E-2	0,000	-2,967E-2	0,000	-3,147E-2	0,000	-3,251E-2	0,000	-3,326E-2	0,000	…	
β_2	2,598E-4	0,000	2,597E-4	0,000	2,423E-4	0,000	2,548E-4	0,000	2,629E-4	0,000	2,664E-4	0,000	…	
	2005		2006		2007		2008		2009		2010		2011	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p
β_0	1,178E+0	0,000	1,185E+0	0,000	1,082E+0	0,000	1,047E+0	0,000	1,041E+0	0,000	1,021E+0	0,000	1,020E+0	0,000
β_1	-3,664E-2	0,000	-3,663E-2	0,000	-3,353E-2	0,000	-3,245E-2	0,000	-3,227E-2	0,000	-3,165E-2	0,000	-3,156E-2	0,000
β_2	2,902E-4	0,000	2,881E-4	0,000	2,648E-4	0,000	2,563E-4	0,000	2,549E-4	0,000	2,501E-4	0,000	2,488E-4	0,000

Table 1. Estimated parameters for Czech male's polynomial regressions of the 2^{nd} order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	…	
β_0	1,106E+0	0,000	1,195E+0	0,000	1,154E+0	0,000	1,018E+0	0,000	1,077E+0	0,000	1,058E+0	0,000	…	
β_1	-3,464E-2	0,000	-3,720E-2	0,000	-3,596E-2	0,000	-3,197E-2	0,000	-3,357E-2	0,000	-3,293E-2	0,000	…	
β_2	2,742E-4	0,000	2,924E-4	0,000	2,833E-4	0,000	2,538E-4	0,000	2,642E-4	0,000	2,588E-4	0,000	…	
	1999		2000		2001		2002		2003		2004		…	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	…	
β_0	1,075E+0	0,000	1,115E+0	0,000	9,969E-1	0,000	1,026E+0	0,000	1,103E+0	0,000	1,039E+0	0,000	…	
β_1	-3,344E-2	0,000	-3,454E-2	0,000	-3,112E-2	0,000	-3,194E-2	0,000	-3,414E-2	0,000	-3,217E-2	0,000	…	
β_2	2,625E-4	0,000	2,699E-4	0,000	2,452E-4	0,000	2,507E-4	0,000	2,664E-4	0,000	2,511E-4	0,000	…	
	2005		2006		2007		2008		2009		2010		2011	
par.	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p	est.	p
β_0	1,152E+0	0,000	1,021E+0	0,000	1,620E+0	0,000	1,622E+0	0,000	1,033E+0	0,000	9,985E-1	0,000	9,735E-1	0,000
β_1	-3,540E-2	0,000	-3,142E-2	0,000	-4,935E-2	0,000	-4,934E-2	0,000	-3,167E-2	0,000	-3,056E-2	0,000	-2,979E-2	0,000
β_2	2,739E-4	0,000	2,437E-4	0,000	3,776E-4	0,000	3,766E-4	0,000	2,445E-4	0,000	2,356E-4	0,000	2,296E-4	0,000

Table 2. Estimated parameters for Czech female's polynomial regressions of the 2^{nd} order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	-2,104E+0	0,015	-2,143E+0	0,029	-2,634E+0	0,008	-2,602E+0	0,001	-2,168E+0	0,033	-1,091E+0	0,301	…	
β_1	9,805E-2	0,007	9,926E-2	0,017	1,205E-1	0,004	1,193E-1	0,001	1,002E-1	0,020	5,358E-2	0,227	…	
β_2	-1,543E-3	0,003	-1,552E-3	0,008	-1,858E-3	0,002	-1,841E-3	0,000	-1,561E-3	0,010	-8,937E-4	0,149	…	
β_3	8,321E-6	0,001	8,320E-6	0,003	9,783E-6	0,001	9,672E-6	0,000	8,313E-6	0,004	5,148E-6	0,074	…	
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	-1,550E+0	0,200	-2,199E+0	0,023	-1,733E+0	0,010	-1,683E+0	0,124	-1,583E+0	0,055	-2,049E+0	0,008	…	
β_1	7,512E-2	0,140	1,024E-1	0,012	8,196E-2	0,005	8,073E-2	0,080	7,689E-2	0,028	9,706E-2	0,003	…	
β_2	-1,226E-3	0,085	-1,606E-3	0,005	-1,307E-3	0,001	-1,303E-3	0,044	-1,256E-3	0,011	-1,543E-3	0,001	…	
β_3	6,831E-6	0,040	8,579E-6	0,002	7,125E-6	0,000	7,161E-6	0,018	6,982E-6	0,003	8,318E-6	0,000	…	
	2005		**2006**		**2007**		**2008**		**2009**		**2010**		**2011**	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**
β_0	-3,127E+0	0,000	-3,802E+0	0,000	-3,367E+0	0,000	-3,017E+0	0,000	-3,521E+0	0,000	-2,778E+0	0,000	-3,563E+0	0,000
β_1	1,438E-1	0,000	1,724E-1	0,000	1,530E-1	0,000	1,379E-1	0,000	1,590E-1	0,000	1,276E-1	0,000	1,605E-1	0,000
β_2	-2,215E-3	0,000	-2,614E-3	0,000	-2,324E-3	0,000	-2,108E-3	0,000	-2,400E-3	0,000	-1,960E-3	0,000	-2,418E-3	0,000
β_3	1,152E-5	0,000	1,334E-5	0,000	1,190E-5	0,000	1,087E-5	0,000	1,221E-5	0,000	1,016E-5	0,000	1,226E-5	0,000

Table 3. Estimated parameters for Czech male's polynomial regressions of the 3rd order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	-2,241E+0	0,000	-2,733E+0	0,000	-2,774E+0	0,000	-1,717E+0	0,003	-2,974E+0	0,000	-2,900E+0	0,000	…	
β_1	1,057E-1	0,000	1,275E-1	0,000	1,287E-1	0,000	8,268E-2	0,001	1,362E-1	0,000	1,330E-1	0,000	…	
β_2	-1,674E-3	0,000	-1,994E-3	0,000	-2,002E-3	0,000	-1,338E-3	0,000	-2,093E-3	0,000	-2,044E-3	0,000	…	
β_3	8,957E-6	0,000	1,051E-5	0,000	1,051E-5	0,000	7,318E-6	0,000	1,084E-5	0,000	1,059E-5	0,000	…	
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	-2,967E+0	0,000	-3,358E+0	0,001	-1,662E+0	0,016	-1,682E+0	0,035	-2,246E+0	0,013	-1,965E+0	0,011	…	
β_1	1,360E-1	0,000	1,530E-1	0,001	8,034E-2	0,006	8,160E-2	0,016	1,063E-1	0,006	9,376E-2	0,004	…	
β_2	-2,090E-3	0,000	-2,333E-3	0,000	-1,302E-3	0,002	-1,325E-3	0,006	-1,683E-3	0,002	-1,497E-3	0,001	…	
β_3	1,081E-5	0,000	1,197E-5	0,000	7,115E-6	0,000	7,246E-6	0,001	8,961E-6	0,001	8,038E-6	0,000	…	
	2005		**2006**		**2007**		**2008**		**2009**		**2010**		**2011**	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**
β_0	-3,381E+0	0,000	-2,429E+0	0,000	-5,221E+0	0,000	-5,363E+0	0,000	-3,010E+0	0,000	-3,395E+0	0,000	-3,212E+0	0,000
β_1	1,546E-1	0,000	1,132E-1	0,000	2,374E-1	0,000	2,435E-1	0,000	1,378E-1	0,000	1,536E-1	0,000	1,456E-1	0,000
β_2	-2,364E-3	0,000	-1,764E-3	0,000	-3,603E-3	0,000	-3,688E-3	0,000	-2,108E-3	0,000	-2,321E-3	0,000	-2,206E-3	0,000
β_3	1,213E-5	0,000	9,231E-6	0,000	1,830E-5	0,000	1,869E-5	0,000	1,082E-5	0,000	1,176E-5	0,000	1,120E-5	0,000

Table 4. Estimated parameters for Czech female's polynomial regressions of the 3rd order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	2,346E-1	0,000	2,485E-1	0,001	2,423E-1	0,000	2,789E-1	0,000	1,468E-1	0,026	2,679E-1	0,003	…	
β_1	-7,936E-3	0,000	-8,205E-3	0,000	-8,061E-3	0,000	-9,039E-3	0,000	-5,299E-3	0,005	-8,759E-3	0,001	…	
β_2	7,325E-5	0,000	7,386E-5	0,000	7,314E-5	0,000	7,938E-5	0,000	5,300E-5	0,000	7,760E-5	0,000	…	
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**	
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**	
β_0	2,344E-1	0,001	2,227E-1	0,002	2,502E-1	0,002	2,274E-1	0,000	1,478E-1	0,001	1,474E-1	0,002	…	
β_1	-7,698E-3	0,000	-7,335E-3	0,000	-8,023E-3	0,001	-7,441E-3	0,000	-5,139E-3	0,000	-5,188E-3	0,000	…	
β_2	6,911E-5	0,000	6,627E-5	0,000	7,022E-5	0,000	6,629E-5	0,000	4,975E-5	0,000	5,042E-5	0,000	…	
	2005		**2006**		**2007**		**2008**		**2009**					
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**				
β_0	2,652E-1	0,000	1,060E-1	0,002	2,168E-1	0,000	1,689E-1	0,000	2,185E-1	0,000				
β_1	-8,572E-3	0,000	-3,987E-3	0,000	-7,099E-3	0,000	-5,721E-3	0,000	-7,019E-3	0,000				
β_2	7,458E-5	0,000	4,175E-5	0,000	6,331E-5	0,000	5,326E-5	0,000	6,142E-5	0,000				

Table 5. Estimated parameters for Slovak male's polynomial regressions of the 2nd order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	2,710E+0	0,000	2,786E+0	0,000	2,797E+0	0,000	2,514E+0	0,000	2,440E+0	0,000	2,425E+0	0,000	…
β_1	-8,319E-2	0,000	-8,534E-2	0,000	-8,576E-2	0,000	-7,734E-2	0,000	-7,540E-2	0,000	-7,506E-2	0,000	…
β_2	6,417E-4	0,000	6,565E-4	0,000	6,607E-4	0,000	5,982E-4	0,000	5,859E-4	0,000	5,841E-4	0,000	…
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	2,387E+0	0,000	2,553E+0	0,000	2,144E+0	0,000	2,294E+0	0,000	2,576E+0	0,000	2,432E+0	0,000	…
β_1	-7,389E-2	0,000	-7,896E-2	0,000	-6,692E-2	0,000	-7,117E-2	0,000	-7,950E-2	0,000	-7,534E-2	0,000	…
β_2	5,750E-4	0,000	6,134E-4	0,000	5,253E-4	0,000	5,551E-4	0,000	6,160E-4	0,000	5,861E-4	0,000	…
	2005		**2006**		**2007**		**2008**		**2009**				
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**			
β_0	3,245E+0	0,000	2,985E+0	0,000	3,362E+0	0,000	3,287E+0	0,000	3,451E+0	0,000			
β_1	-9,894E-2	0,000	-9,134E-2	0,000	-1,022E-1	0,000	-9,974E-2	0,000	-1,044E-1	0,000			
β_2	7,558E-4	0,000	7,006E-4	0,000	7,787E-4	0,000	7,580E-4	0,000	7,908E-4	0,000			

Table 6. Estimated parameters for Slovak female's polynomial regressions of the 2^{nd} order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	8,618E-2	0,883	-7,415E-2	0,921	-5,242E-1	0,262	-7,311E-1	0,317	-4,247E-1	0,539	-1,591E+0	0,061	…
β_1	-1,715E-3	0,944	5,321E-3	0,865	2,407E-2	0,220	3,330E-2	0,277	1,866E-2	0,519	6,917E-2	0,052	…
β_2	-1,311E-5	0,969	-1,139E-4	0,793	-3,729E-4	0,172	-5,084E-4	0,232	-2,796E-4	0,486	-1,004E-3	0,043	…
β_3	3,971E-7	0,800	8,633E-7	0,665	2,051E-6	0,105	2,702E-6	0,169	1,529E-6	0,408	4,974E-6	0,030	…
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	-7,000E-1	0,293	-9,078E-1	0,189	-1,080E+0	0,162	-6,125E-1	0,281	-7,591E-2	0,864	1,017E-1	0,834	…
β_1	3,147E-2	0,259	4,005E-2	0,167	4,775E-2	0,140	2,777E-2	0,243	4,240E-3	0,819	-3,270E-3	0,872	…
β_2	-4,746E-4	0,221	-5,916E-4	0,142	-7,041E-4	0,118	-4,225E-4	0,202	-8,045E-5	0,755	2,380E-5	0,933	…
β_3	2,500E-6	0,163	3,025E-6	0,105	3,560E-6	0,087	2,247E-6	0,142	5,986E-7	0,614	1,224E-7	0,925	…
	2005		**2006**		**2007**		**2008**		**2009**				**…**
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**			**…**
β_0	-6,641E-1	0,032	1,735E-1	0,604	-2,127E-1	0,448	-5,690E-1	0,066	-4,416E-2	0,920			…
β_1	3,038E-2	0,020	-6,818E-3	0,626	1,091E-2	0,353	2,521E-2	0,053	3,993E-3	0,828			…
β_2	-4,662E-4	0,011	8,105E-5	0,675	-1,866E-4	0,254	-3,762E-4	0,038	-9,145E-5	0,720			…
β_3	2,486E-6	0,004	-1,807E-7	0,839	1,149E-6	0,131	1,974E-6	0,020	7,028E-7	0,550			…

Table 7. Estimated parameters for Slovak male's polynomial regressions of the 3^{rd} order. Source: author's calculations and illustrations.

	1993		1994		1995		1996		1997		1998		…
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	-9,869E+0	0,000	-1,043E+1	0,000	-1,008E+1	0,000	-8,254E+0	0,000	-7,401E+0	0,000	-6,180E+0	0,000	…
β_1	4,441E-1	0,000	4,688E-1	0,000	4,541E-1	0,000	3,740E-1	0,000	3,371E-1	0,000	2,857E-1	0,000	…
β_2	-6,678E-3	0,000	-7,036E-3	0,000	-6,833E-3	0,000	-5,668E-3	0,000	-5,141E-3	0,000	-4,424E-3	0,000	…
β_3	3,366E-5	0,000	3,537E-5	0,000	3,445E-5	0,000	2,881E-5	0,000	2,633E-5	0,000	2,302E-5	0,000	…
	1999		**2000**		**2001**		**2002**		**2003**		**2004**		**…**
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**…**
β_0	-5,920E+0	0,004	-5,440E+0	0,007	-1,648E+0	0,114	-3,487E+0	0,143	-5,145E+0	0,030	-2,982E+0	0,345	…
β_1	2,743E-1	0,002	2,561E-1	0,003	9,205E-2	0,039	1,711E-1	0,089	2,442E-1	0,015	1,516E-1	0,254	…
β_2	-4,259E-3	0,001	-4,038E-3	0,001	-1,682E-3	0,008	-2,809E-3	0,047	-3,877E-3	0,007	-2,564E-3	0,167	…
β_3	2,223E-5	0,000	2,139E-5	0,000	1,015E-5	0,001	1,547E-5	0,019	2,066E-5	0,002	1,448E-5	0,093	…
	2005		**2006**		**2007**		**2008**		**2009**				**…**
par.	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**	**est.**	**p**			**…**
β_0	-9,805E+0	0,000	-7,360E+0	0,000	-1,027E+1	0,000	-1,245E+1	0,000	-1,308E+1	0,000			…
β_1	4,481E-1	0,000	3,423E-1	0,000	4,691E-1	0,000	5,601E-1	0,000	5,884E-1	0,000			…
β_2	-6,839E-3	0,000	-5,319E-3	0,000	-7,152E-3	0,000	-8,402E-3	0,000	-8,827E-3	0,000			…
β_3	3,492E-5	0,000	2,768E-5	0,000	3,646E-5	0,000	4,212E-5	0,000	4,422E-5	0,000			…

Table 8. Estimated parameters for Slovak female's polynomial regressions of the 3^{rd} order. Source: author's calculations and illustrations.

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