

DERAS VERSUS MS EXCEL SOLVER IN LEVELLING THE LIFE EXPECTANCY AT BIRTH

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Abstract. The article deals with using of MS Excel Solver for demographic calculations. We will focus on the levelling of specific mortality rates for higher ages. The reason is that there is a gradual improvement in mortality. Therefore, it is more and more important to best capture the mortality of oldest persons. In our study, we will focus on levelling specific mortality rates by Gompertz–Makeham (and modified Gompertz–Makeham) function. An important part of the article will provide the process for estimating the parameters of mentioned functions using the procedure of MS Excel Solver. Our goal is to present the methodology that we use for estimating parameters of these functions. The procedure will be applied on data about mortality of Czech population. The results will be compared with the estimates, obtained from the software DeRaS, which is used for estimating parameters of selected models used for smoothing specific mortality rates. The aim is to compare the results obtained from both procedures. There will be presented the possible consequences of current development of mortality in the conclusion.

Key words. DeRaS, MS Excel, Life Expectancy at Birth, Gompertz–Makeham function

Mathematics Subject Classification: 90C30

1 Introduction

Recently, there is a lengthening of the human life in the Czech Republic and other developed and developing European countries (see e.g. Arltová et al. [1]). Therefore, it is increasingly important to have the best vision of the mortality trend for oldest people (Lagerås [13] or Gavrillov, Gavrilova [10]). For special purposes of demographic applications it is necessary to estimate and refine the parameters of specific functions. One of these function is Gompertz–Makeham function, using for levelling specific mortality rates (see Gompertz [11], Makeham [15]), which is used in the construction of complete life tables for commercial use (see e.g. the projection of the human capital of the Czech Republic and its regions to 2050 by Fiala et al. [9] or the development of life expectancy in the Czech Republic and EU countries by Langhamrová Jana and Langhamrová Jitka [14]). This function is of exponential type and therefore is effective use a special procedure for its solution. The initial estimates of its parameters can be calculated from empirical data (see Cipra [5]) and using the optimization software is also possible to improve these estimates. Given to the different development of mortality rates at the highest ages compared to younger ones, it is necessary to balance them. In the past, the most used model was the Gompertz–Mahekam (or

modified Gompertz–Makeham model, see e.g. Burcin et al. [3]). Recently exist the discussions about the suitability of these models light of changes in mortality trend (Gavrilov, Gavrilova [10]). For some populations is still true that Gompertz–Makeham model is the best.

2 Methodology

For the analysis of mortality is most commonly used the life expectancy at exact age x , which determines how long will person lives in average. Calculating of life tables proceed in several steps. The first step is calculation of specific mortality rates (see e.g. Fiala [8] or Koschin [12])

$$m_x = \frac{M_x}{S_x}, \quad (1)$$

where M_x is the number of deaths at age x , S_x is the number of persons at age x . In the next part we calculate the probability of dying, the probability of survival and biometric measures for the table population. The last indicator is life expectancy at exact age x .

During the calculating of the specific mortality rates is important to observe their development at the highest ages (see e.g. Ekonomov, Yarigin [6]). As was already mentioned, that the mortality of older people (approximately from 60 years) is different from younger ones. For this reason, it is good to use some of the existing models, which are used for levelling mortality curves. Among the most frequently used models is included Gompertz–Makeham function (G–M function). Later it was followed by the modified version of Gompertz–Makeham function (mG–M function). Let us explain the G–M function (see e.g. Boleslawski, Tabeau [2]) as

$$\mu_x = a + bc^x, \quad (2)$$

where μ_x is the intensity of mortality, x is age and a , b , and c are parameters. The function is based on the assumptions that the increments of the intensity of mortality are still the same with increasing age. Given that this assumption is not possible to apply it from 85 years and more, therefore it was designed mG–M model. It assumes that the increments of the intensity of mortality decrease with increasing age.

For the intensity of mortality is true the equation:

$$\mu\left(x + \frac{1}{2}\right) \doteq m_x, \text{ where } x = 1, 2, \dots, 59. \quad (3)$$

For higher ages is used for the levelling and extrapolating mortality curve the Gompertz–Makeham function. In our study, we will present two possible ways for estimating of unknown parameters of G–M and mG–M function. In the first way will be used the Solver in MS Office Excel (version 2010, for more information see e.g. Šimpach [16]). We have to find the initial estimates of parameters in the first step and we will improve them using by the ordinary least squares method (OLS). Because we need to estimate three parameters, we will need three equations. It is important to choose the beginning of the first interval x_0 (i.e. the age from which will be performed the equalization) and set the length of the interval k . After that, we will calculate the sum of empirical mortality rates in the three intervals:

$$G_1 = \sum_{x=x_0}^{x_0+k-1} m_x, \quad G_2 = \sum_{x=x_0+k}^{x_0+2k-1} m_x \quad \text{and} \quad G_3 = \sum_{x=x_0+2k}^{x_0+3k-1} m_x, \quad (4)$$

where $k = 5$ and $x_0 = 60$. The value of G_1 we can also express using by model parameters and age x as

$$G_1 = \sum_{x=x_0}^{x_0+k-1} (a + b.c^{x+\frac{1}{2}}). \quad (5)$$

In the same way, we could expressed G_2 and G_3 . By subtracting and dividing of the individual equations we exclude the most of the parameters and we get

$$c^k = \frac{G_3 - G_2}{G_2 - G_1}. \quad (6)$$

The value of the parameter c we get as k -(th) square root of the c^k

$$c = \sqrt[k]{c^k}. \quad (7)$$

To estimate the initial values of a and b we have to calculate the value of sub-expression of K_c as

$$K_c = c^{x_0 + \frac{1}{2} \cdot \frac{c^k - 1}{c - 1}}, \quad (8)$$

and the parameters a and b we obtain as

$$b = \frac{G_2 - G_1}{K_c \cdot (c^k - 1)}, \quad (9)$$

respectively

$$a = \frac{G_1 - b \cdot K_c}{k}. \quad (10)$$

The last step is the calculation of the weighted squared deviations (WSD), through which we will optimize the parameter of G–M (respectively mG–M) model

$$WSD = \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2 \quad \text{for } x \leq 60; y >, \quad (11)$$

where $S_{t,x}$ is the number of living at age x in year t , $S_{t+1,x}$ is the number of living at age x in year $t+1$, m_x are specific mortality rates, $\tilde{m}_x^{(GM)}$ are levelled values of specific mortality rates using by G–M or mG–M model, y is the highest age for which we have a non-zero value of m_x . When we optimize the parameters using by the OLS, it is necessary to create two instrumental sums,

$$S_1 = \sum_{x=60}^{82} \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2, \quad (12)$$

respectively

$$S_2 = \sum_{x=83}^y \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2, \quad (13)$$

which will help us to optimize the initial values of parameters. For continuation of analysis, let us explain mG–M function (see e.g. Thatcher et al. [17]) as

$$\mu_x = a + bc^{x_0 + \frac{1}{\gamma} \ln[\gamma(x-x_0)+1]}, \quad (14)$$

where a , b and c are the parameters of the original G–M function, x_0 is the age from which will be performed equalization and γ is a parameter of mG–M function.

For estimating of model parameters we can use a specialized software designed for this purposes (e.g. DeRaS by Burcin et al. [4]) or we can use the Solver in MS Office Excel. An indisputable advantageous of the calculation in MS Excel is that it is easily accessible for everybody and cheaper, (see Šimpach [16]), unlike the special software. The procedure in the Solver is quite easy. As “Set Objective” we select either S_1 or S_2 and we will minimize it. Situation depends on whether we optimize the parameters of G–M or mG–M model. As “Changing variable cells” we set the initial estimates of parameters a , b , and c (for G–M model), or the value of the parameter γ (for mG–M model). Because we do not have an initial value for γ , we set it to 0.02 (see e.g. recommendations by Fiala [7]). This value will be improved.

DeRaS used for estimating of unknown parameters from the G–M function is used nonlinear regression procedure using iteratively reweighted least squares.

3 Results

In our study we focused on the analysis of the mortality trend of the Czech population from 1970 to 2009. The results obtained from the analysis will be published separately for males and females. From the Fig. 1 and Fig. 2 is clear, that the life expectancy at birth increased during the reporting period. For males, its value has changed more than for females. For males increased from 66 years to 74, while for females increased from 73 to 80 years.

If we closer examine the results, we will find out that the values of estimated life expectancy at birth by G–M model obtained from DeRaS and MS Excel Solver are very similar. The difference occurs except for the second decimal place. For mG–M conclusions are similar. This means that the results obtained from both methods differ by approximately at the second decimal place. If we assess which model provides higher (or lower) values of life expectancy at birth, we will find out, that there is no clear conclusion (during the monitored period it become to changes).

From the results obtained according to both methods we can make a similar conclusion like for men. The results obtained by GM model according to both methods are very similar (results, during the reference period differs about 0.1). On the contrary, the values of life expectancy according to mG–M model differ only in the order of hundredths.

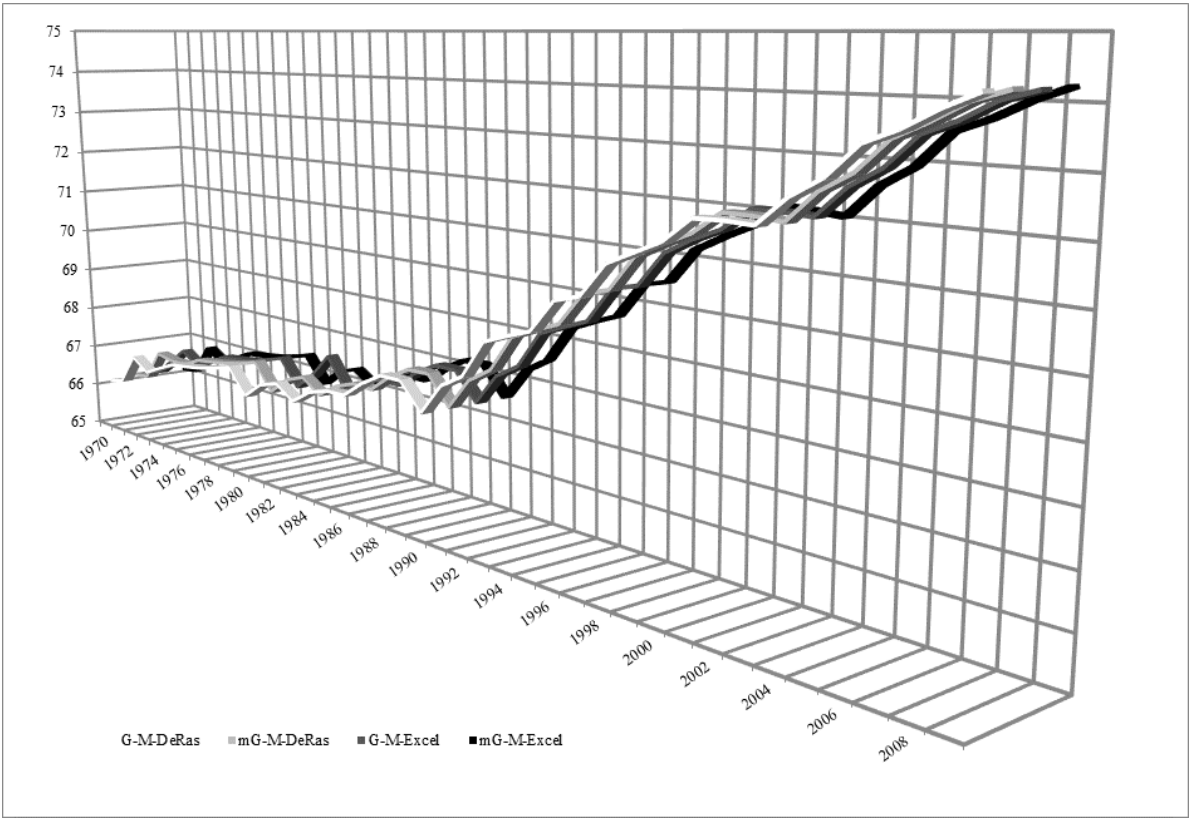


Fig. 1. Levelled life expectancy at birth for males. Source: authors' calculations and illustration

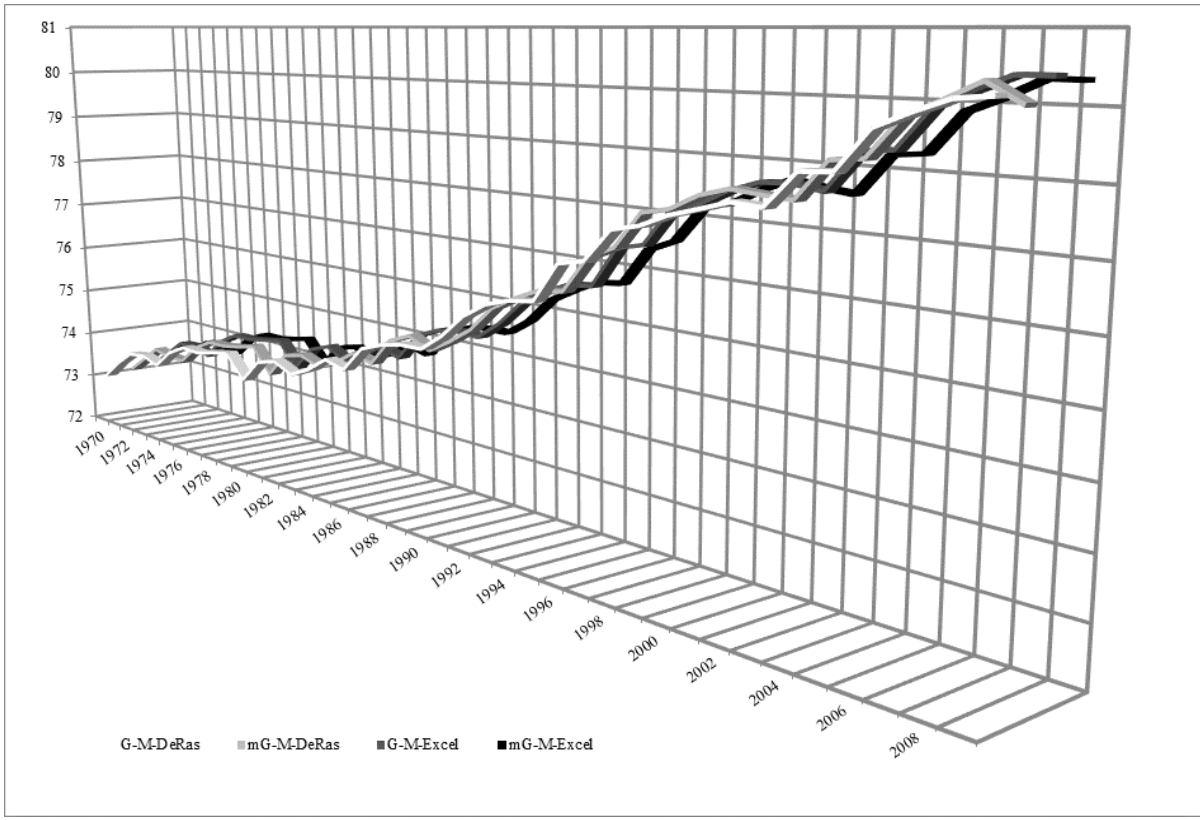


Fig. 2. Levelled life expectancy at birth for females. Source: authors' calculations and illustration

4 Conclusion

From the obtained graphical outputs is clear that the life expectancy at birth for both sexes increased. It means that people live longer in average than it did about 30 years ago. If we focus on results obtained by using both methods, we find out that the values of life expectancy obtained from the software DeRaS and MS Excel Solver differ only marginally. It does not mean anything more than that the MS Excel Solver provides relatively appropriate estimations of life expectancy and therefore it is good substitute if we do not have a special soft-ware. The results obtained according to both methods will differ slightly on second decimal place.

Acknowledgement

The authors gratefully acknowledge the Scientific Grant Agency IGA VŠE for supporting this work under the Grant No. 24/2013 “The mortality and aging population of the Czech Republic”.

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